ANALYSIS FLUXIONUM.

AUCTORE GUIL. HALES, D.D.

RECTORE DE KILLESANDRA,

ET NUPER TRIN. COLL. DUBLIN SOCIO, AC LINGUARUM ORIENTALIUM PROFESSORE.

Sanctos ausus recludere fontes. VIRG.

LONDINI:

TYPIS J. DAVIS, IN VICO DICTO CHANCERY-LANE;

PROSTAT APUD JOHANNEM WHITE, BIBLIOPOLAM, IN VICO DICTO FLEET-STREET.

MDCCC.

INDEX CAPITUM.

PARS PRIMA.

§ 1. *DE Fluxionum Inventione* - - - - - - - Pag. 1
§ 14. *De Objectionibus hactenùs Impugnantium Evidentiam et Certitudinem Methodi Fluxionum* - - - - - - - - 6
§ 28. *De Methodo Rationum Primarum et Ultimarum, seu Limitum* - - 15
§ 39. *Colson's Account of the Method of Fluxions* - - - - 19
§ 43. *Maclaurin's Account of the Method of Fluxions* - - - - 21
§ 47. *De Applicatione Methodi Rationum Primarum et Ultimarum, ad Quadraturam Curvarum, atque ad Mechanicam Rationalem* - - 23
§ 64. *De Fluxionum Elementis Primis* - - - - - - 28

PARS SECUNDA.

§ 78. *Definitiones* - - - - - - - - - - - Pag. 35
§ 79. *Axiomata* - - - - - - - - - - 36
§ 80. *De Notatione Fluxionali* - - - - - - - 37
§ 83. *De Methodo Fluxionum Directâ* - - - - - - 38
§ 102. *De Fluxionibus Superiorum Ordinum* - - - - - - 47
§ 106. *De Theoremate Binomio* - - - - - - - 49
§ 112. *De Methodo Fluxionum Inversâ* - - - - - - 52
§ 118. *De Correctione Fluentium* - - - - - - - 53
§ 119. *De Fluxionibus diversas Fluentes admittentibus* - - - - 54

§ 120. *De Fluxionibus Logarithmorum* - - - - - Pag. 55
§ 128. *De Fluxionibus Quantitatum Exponentialium* - - - - 57
§ 129. *De Logarithmis Imaginariis* - - - - - - - 58
§ 132. *Formulæ Fluxionum atque Fluentium* - - - - - 59
§ 133. *De Formularum Applicatione* - - - - - - 61

APPENDIX PRIMA.

§ 135. *De Analysi Antiquâ* - - - - - - - Pag. 65

APPENDIX SECUNDA.

§ 142. *De Æthere Vibratorio* - - - - - - - Pag. 73
§ 151. *De Sensorio, et de Vi Sentiendi* - - - - - - 79
§ 155. *Maclaurin's Last Words* - - - - - - - - 82
§ 157. *D'Alembert's Mournful Reflexions* - - - - - - 85
§ 162. *De Philosophiâ Atomicâ Priscorum Chaldæorum et Hebræorum* - 88
§ 174. *De Æthere Antiquo, Deo Ficto Gentilium* - - - - 93
§ 196. De Ente Supremo - - - - - - - - 102
De Nominibus Dei Primitivis, Not. (*m*) - - - ib.

FRANCISCO MASERES,

CURIÆ SCACCARII REGIS MAGNÆ BRITANNIÆ IN ANGLIÂ BARONI QUINTO;

ANALYSTÆ EGREGIO,

MATHESEOS PATRONO,

VIRO BONO,

AMICO FIDO;

VINDICIAS HÁSCE NEWTONIANAS—

METAPHYSICAS ET MATHEMATICAS,

PHYSICAS ET THEOLOGICAS;—

QUÂ PAR EST OBSERVANTIÂ,

GRATUS LUBENSQUE

D. D. D.

GUILIELMUS HALES.

PRÆFATIO.

INTER THEORIAM ÆQUATIONUM ALGEBRAÏCARUM *atque* FLUXIONALIUM *tanta intercedit Analogia, eædémque tam arctâ affinitate, imò cognatione, inter sese consociantur et conjunguntur; ut absque hujus ope, illa jure imperfecta, et quasi manca, existimari debeat: Nam per calculum* FLUXIONALEM, *seu, ut ab exteris designatur,* DIFFERENTIALEM, *(vix nisi nomine et notatione à se invicèm discrepantes) optimè promovenda est* ALGEBRA. *Sicut unico patere poterit exemplo,* THEOREMATIS BINOMII, *quod, licet indoles ejus sit omninò fluxionalis, omnes tamen* MATHESEOS *regiones permeat et adauget. Hanc igitùr* ANALYSIN FLUXIONUM, ANALYSI ÆQUATIONUM *pridem editæ* (*), *planè gemellam, edere jam libet; ut pro virili fungar munere interpretis fidi* ANALYSEOS ANTIQUÆ *illius, veræ et genuinæ* :—" Quæ cum *Algebrâ Speciosâ* facilitate contendit, evidentiâ verò et demonstrationum elegantiâ eam longè superare videtur;"—*Teste, peritissimo Geometrâ simùl ac Analystâ,* EDMUNDO HALLEY; *qualem eam optimè exponit* PAPPUS ALEXANDRINUS *(in* APPENDICE, I. § 136.) *ex scriptis Veterum Analystarum* :—ARCHIMEDIS, in Quadraturâ *Parabolæ*; APOLLONII, in tractatu de Sectione *Rationis*; EUCLIDIS, in Libris *Datorum* et *Porismatum*; ERATOSTHENIS, de Mediis *Proportionalibus*, &c.—*Hanc verò* ANALYSIN ANTIQUAM *pænè amissam,* (CARTESIO, *ejúsque sectatoribus, haud minùs* MATHESIN *quàm* PHILOSOPHIAM *inquinantibus, adhibendo demonstrationes, antiquis quidem breviores,*

(*) *Tractatum priorem* (ANALYSIN ÆQUATIONUM) *impensis propriis edidi anno* 1784; *hunc verò, pro singulari suo in* MATHESIN, PHILOSOPHIAMQUE BRITANNICAM *studio, impensis suis editurus est vir cl.* FR. MASERES, *vel ipso flagitante. Neque, ut spero, in amici dignissimi modestiæ peccâsse censebor, si in lucem, (etiam illo invito), excerpta quædam deprompserim ex epistolâ ejus, hanc rem spectantia; Sept.* 17. *anno* 1799 *datâ* :

—" I am glad to find that you have been so actively and usefully employed, since your return to IRELAND, as in revising and enlarging your *Analysis Fluxionum*; which I hope you will soon publish. If you will send it to me, I will print it for you, either in the *Fifth* Volume of the *Scriptores Logarithmici*, or by itself;—or half the number of copies in the *Scriptores Logarithmici*, and the other half by themselves, as you shall direct. And I believe that last manner of printing it would be the best :—that is, to print 250 copies in the *Scriptores Logarithmici*, and another 250 separately; as, by this way, it will come *sooner* into the hands of the Learned."——

—" I wish you every opportunity of propagating your taste for sound literature and science; and thereby of becoming *eminently useful* to your COUNTRY: than which, *nothing can afford* you *greater satisfaction*."

See § 4, Note (*a*); § 17, Note (*d*); and § 131.

at longè laxiores atque infirmiores) curiosâ felicitate restituit, et instauravit et perfecit NEWTONUS, *Veterum evidentiam et certitudinem, Recentiorum brevitatem et facilitatem, rarissimâ conjunctione assecutus; si modò rite expendatur et probè intelligatur imperatoria brevitas quâ* METHODUM FLUXIONUM *delibando transcurrit Inventor ille sagacissimus.*

Hujus Methodi fons et origo est Doctrina RATIONUM PRIMARUM ET ULTIMARUM; *cujus ope demonstrantur* LEMMATA *illa præclara seu* PRINCIPIA MATHEMATICA, *perbrevia et subtilissima, in primâ sectione doctissimi et celeberrimi illius libri cui titulus,* Philosophiæ Naturalis Principia Mathematica. *Hæc verò Doctrina, qualem ipse auctor eam orbi litterato in diversis suis scriptis proposuit, pulcherrimam exhibet synopsin paritèr et extensionem* METHODI EXHAUSTIONUM *à veteribus geometris usurpatæ et excultæ, quam vir mirandâ sagacitate præditus uno eodémque tempore in compendium redegit et ad novas et maximè arduas speculationes felicissimè promovit et applicuit.*

Cæterùm fatendum est hanc Doctrinam RATIONUM PRIMARUM ET ULTIMARUM, *ex intimis* METAPHYSICÆ *fontibus haustam, et* "quâ potuit brevitate" *à* NEWTONO *traditam, obscuritate non facilè discutiendâ involvi: quocircà expositio ejusdem, satìs lucida, evidens, et accurata (non obstantibus tot tantísque Interpretibus Analyseos Newtonianæ) hactenùs desiderari videtur; éstque conditio sine quâ non clarè cernere atque animo complecti licebit fundamentum solidum et nullis cavillationibus dimovendum, cui adstruitur tota* THEORIA FLUXIONUM, *neque diluere objectiones doctorum quorundam virorum, qui etiàm his seris temporibus ejusdam evidentiam et certitudinem impugnant, sive ex inscitiâ vel imperitiâ, vel denique ex invidiâ et partium quodam studio, oriundas.*

Itaque, ne falsis criminibus ulteriùs premi videatur Methodus Fluxionum, in PRIMA PARTE, *ausus sum fontes ipsos* METAPHYSICOS *recludere ex quibus derivantur* PRINCIPIA MATHEMATICA FLUXIONUM, *in* SECUNDA PARTE; *Analyseos Antiquæ et veræ quasi custos rigidúsque Satelles; et magni mirandíque* ANALYSTÆ BRITANNICI *æquus et candidus vindex; excussis quàm fidelissimè et quàm accuratissimè potuerim, objectionibus Analystarum magni nominis,* BERKELEY, LANDEN, D'ALEMBERT, TORELLI, DE LA GRANGE, &c. et nuperrimè, CENSORIS (ut ferunt, *Professorii* et *Academici,)* apud THE MONTHLY REVIEW, *Appendix, April* 1798, *Vol.* 28. N. S.

In hâc Disquisitione, subtilissimâ sane et reconditissimâ, et inter laborem "tardè eruendi quæ tam altè jacent," *Demonstrationem Theorematis Binomii, à doctissimo viro, Johanne Landen, inventam, (quam ego dudùm admiratus fueram et in* ANALYSI ÆQUATIONUM *adhibueram et excolueram,)*

eram,) iterùm ad examen revocare visum est; et, cum ista demonstratio vix, et ne vix quidem, in principiis in quibus fundatur, à computo fluxionali discrepare videtur, aliam demonstrationem ejusdem Theorematis omninò, novam, et in principiis Methodi Fluxionum fundatam, huic Landenianæ substituendam esse judicavi; quæ quidem est ipsâ longè facilior, et simplicior, et ad naturam Theorematis demonstrandi magìs accommodata, utpote quæ derivatur ex analogiâ quâdam elegantissimâ, quæ intercedere observatur inter SERIEM BINOMIAM *et* SERIEM FLUXIONUM ORDINUM SUPERIORUM, *quarum serierum termini correlativi sunt ad se invicèm semper in datâ quâdam ratione.*

His VINDICIIS NEWTONIANIS—METAPHYSICIS *et* MATHEMATICIS—in APPENDICE II. *alias quoque*—PHYSICAS *et* THEOLOGICAS—*subjungere visum est; ut laus debita illi* PHILOSOPHO CHRISTIANO, *integra servetur et illibata; quam inquinare non verentur, falsò atque ignavè,* MINUTI *quidam* PHILOSOPHI, *hoc* "SÆCULO *perperàm sanè ab iis, eorúmque sectatoribus,* RATIONIS *dicto. Pudétque inter hujusmodi obtrectatores, inveniri ipsius* NEWTONIANÆ PHILOSOPHIÆ (ad THEOLOGIAM *sobriè et humilitèr hactenùs ancillantis,) non solùm Discipulum sed etiàm Professorem,* ROBISON, *apud* EDINBURGENSES; *temerè inculpantem theoriam illam sublimem at longè subtilissimam* " De pulsibus *medii ætherii*, vibrando propagatis à *corporibus tremulis* ad *sensorium* usque; ibíque, à *substantiâ sensitivâ*, seu *vi sentiendi, intelligendi*, et *agendi* perceptis:"—*cujus objectiones penitiùs et fusiùs, ex* NEWTONI *scriptis, et testimoniis* PHILOSOPHIÆ PRIMARIÆ *et* SACRÆ, *antiquitùs in honore habitæ, in hâc secundâ Appendice excutiuntur.*

Quàm cautè verò, et quàm timidè procedendum in luce sublustri Analyseos sublimioris,—NATURAM DIVINAM HUMANAMQUE *explorantis, ideásque claras et determinatas* INFINITATIS *arripere gestientis,—monet* MACLAURIN *optimus:*

" In reasoning concerning *finite* quantities, WE apprehend that GEOMETRICIANS cannot be too scrupulous in admitting or treating of *Infinites*; of which our ideas are *so very imperfect.*—PHILOSOPHY probably will always have its *mysteries*; but these are to be avoided in GEOMETRY (*), and

(*) GEOMETRY has its *mysteries*, no less than PHILOSOPHY and RELIGION: Witness the elementary doctrine of *Parallels*, resting on the disputed foundation of the *eleventh* axiom of *Euclid*, the intuitive evidence of which is denied by *Tacquet* and others; and the *quadrature of the circle*, which can only be solved by approximation, still remains the reproach of Geometricians. In ALGEBRA, the *irreducible case* of cubick equations exhibits two of the roots under an impossible form, when all are real: which, notwithstanding, can be accurately extracted in some particular cases; and in the rest approximately, by means of the *Binomial Theorem*. And in FLUXIONS, too, the famous *Problem of the Three Bodies*, so necessary to determine the accelerations or retardations of the motions of the planets and the derangements

and we ought to guard against abating from its strictness and evidence the rather, that *an absurd* PHILOSOPHY *is the natural product of a vitiated* GEOMETRY."

KILLESANDRA,
Oct. 23, 1799.

ments of their orbits, by their mutual attractions, (of which approximate solutions have been given by *Euler*, *Clairaut* and *D'Alembert*,) is thus represented by the first of these three great men, with all the candour of a profound Mathematician :—" *Hujus problematis enodatio compleƫta omnes* ANALYSEOS *vires transcendere videtur.*"—The difficulty consists in integrating three differential equations of the second order ; or, in the language of Newton's doctrine of Fluxions, in finding the fluents of three fluxionary equations of the second order.—Indeed the whole theory of the *Resolution of the Higher Equations* in *Algebra*, and the *Inverse Method of Fluxions*, which corresponds thereto, is extremely imperfect still, in this OUR INFANCY OF SCIENCE.—See *Maclaurin*'s dying words, *Appendix*, II. § 162.

ANALYSIS FLUXIONUM.

PARS PRIMA.

DE FLUXIONUM INVENTIONE.

§ 1. *THEORIÆ Fluxionum* præſtantia inſignis, omnis *Matheſeos* fines peragrantis, promoventis et illuſtrantis, prorsùs poſtulare videtur ut præmittantur aliqua, more hiſtorico, de ejus inventione; qualia extant in *Commercio Epiſtolico Johannis Collins* et aliorum, anno Domini 1712, edito, atque iterùm auctiùs anno Domini 1722; in *Raphſon's Hiſtory of Fluxions*; in diverſis editionibus *Principiorum* Newtoni; *Hutton's Mathematical Dictionary*; et *Monthly Review, Append.* Vol. 28, 1799.

2. Utrum *Newtono* an *Leibnitzio* tribuenda ſit laus hujus inventionis, et uter horum eam priùs meditando extuderat; diù atque acritèr conteſtatum eſt inter fautores *calculi fluxionalis* et *calculi differentialis*, vix ab invicèm niſi merâ notatione diſcrepantium; et *adhuc ſub judice lis eſt.*

3. Veruntamen hanc controverſiam penitiùs inſpicienti, et ſine partium ſtudio perpendenti, ſatìs conſtabit, utrumque horum illuſtrium Geometrarum pro ejuſdem inventore jure cenſeri poſſe; utrumque, præcellenti ingenio et mirâ ſagacitate præditum, utrumque eodem "*labore improbo*," et "*cogitandi vi indefeſſâ*" functum, ex iiſdem fontibus ſcientiæ utrique patentibus, tandem eadem *elementa metaphyſica*, eadem *principia mathematica*, methodi ſive *fluxionalis* ſive *differentialis*, hauſiſſe; atque ſectatoribus ſuis delibâſſe, principio, carptìm, ac ſine demonſtrationibus edita. Háſce quidem *Newtonus* poſteà, obitèr tantùm, et quaſi invitus, longo poſt tempore ſupplevit, parcè et "*quâ potuit brevitate*," in *Lemmate Secundo Lib. II. Princip.*; in *Quadraturâ Curvarum*; et in *Analyſi per Æquationes numero terminorum infinitas*:—Hanc curam *Leibnitzius* (vir omnimodæ ſcientiæ avidus, et à Geometriæ ſtudio nimìs ſæpe ad alias ſpeculationes ingenii vires transferens) prorsùs neglexit, diſcipulíſque ſuis celeberrimis, *Jacobo* et *Johanni Bernoulli*, eorúmque ſucceſſoribus *Euler*, *D'Alembert*, *La Grange*, et *La Place*, legavit; qui munus ſibi demandatum, demonſtrandi et in immenſum promovendi ſines fluxionum, longè ultrà limites *Newtonianos* et *Leibnitzianos*, ingenti cum emolumento ipſius MATHESEOS, exſecuti ſunt.

4. Glo-

4. Gloriabatur certè *Leibnitz*,—" ſe *Newtono* nil debuiſſe ;" nec inficiabatur *Newtonus:* pro ſuâ modeſtiâ inexpugnabili nihil ſibi arrogans, et tantùm poſtulans *jus æquum partitionis*, patiens et commodus; " cui omnium mortalium minimè neceſſarium fuit " *laudem aliunde mutuari:—Hanc ſibi ipſi peperit.*" (*a*). Audiamus ipſum, in *primâ editione* PRINCIPIORUM, anno 1687, p. 253. de *Fluxionum inventione* differentem :

" *In literis quæ* MIHI *cum Geometrâ peritiſſimo* GULIELMO, GOTHOFREDO, LEIBNITIO *annis abhinc decèm* [1677] *intercedebant, cum ſignificarem me compotem eſſe methodi determinandi Maximas et Minimas, ducendi Tangentes, et ſimilia peragendi; quæ in terminis ſurdis æquè ac in rationalibus procederet; et literis tranſpoſitis hanc ſententiam continentibus*—" Datâ æquatione, quotcunque fluentes quantitates involvente, fluxiones invenire"—*eandem celarem: reſcripſit vir clariſſimus* ' ſe quoque in ejuſmodi methodum incidiſſe ;' *et methodum ſuam communicavit, à meâ vix abludentem, præterquam in verborum et notarum formulis*," [" et *ideâ generationis quantitatum*"—inſeruntur hæc quatuor verba in editione ſecundâ PRINCIPIORUM anno 1713]—"*Utriuſque fundamentum continetur in hoc lemmate.*" [II. *Lib.* II. PRINCIP.]

5. Et certè fatendum eſt, ſemina non obſcura *methodi ſuæ differentialis*, poſuiſſe *Leibnitzium*, in *Theoriâ Notionum Abſtractarum*, Academiæ Regiæ *Pariſienſi* dicatâ, anno 1671; antequam cum *Newtono* commercium aliquod epiſtolicum inſtituiſſet. Hanc methodum poſteà publici juris fecit, in epiſtolâ quâdam in *Actis Lipſ.* Jan. 21, 1677, editâ, verùm abſque demonſtratione prolatam; quam poſteà ſuppleverunt, celeberrimi fratres, *Daniel et Johannes Bernoulli*, famæ *Leibnitzianæ* vindices acerrimi, iiſdémque fontibus analyſeos ſublimioris haud levitèr imbuti.

(*a*) Dr. Halley, in his letter to Mr. *Newton*, dated May 22, 1686, informing him that the *Royal Society* had reſolved to print his " *Incomparable Treatiſe* intitled PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA," at their own charge, in a large quarto of a fair letter—" judging that ſo excellent a work ought not to have its publication *delayed*"—concludes with ſtating Mr. *Hooke*'s claim —" as having ſome pretenſions upon the invention of *the rule of gravity being reciprocally as the ſquares of the diſtances from the centre.* He ſays you had the notion from him, though he owns the demonſtration of the *curves* generated thereby to be wholly your own.—How much of this is ſo, you know beſt ; as likewiſe, what you have to do in this matter. Only Mr. *Hooke* ſeems to expect you ſhould make ſome mention of him in the Preface, which it is poſſible you may ſee reaſon to prefix."

" I muſt beg your pardon that it is I that ſend you this ungrateful account ; but I thought it my duty to let you know it, that ſo you might act accordingly : *being in myſelf fully ſatisfied, that the greateſt candour imaginable is to be expected from a perſon who has of all men the leaſt need to borrow reputation.* "

See the whole letter, and the proceedings of the Royal Society on this buſineſs, in *Birch's Hiſtory of the Royal Society*, Vol. IV. p. 484.

In conſequence of this unfounded claim of Dr. *Hooke*, and his own averſion to controverſy, *Newton* withdrew the third book of the *Principia*, which treated of the *Syſtem of the World* in a popular way ; wiſhing not to let it go abroad into the world, without a ſtrict and rigorous demonſtration ; as it appeared afterwards :—

" *De hoc argumento compoſueram librum tertium* [DE MUNDI SYSTEMATE] *methodo populari, ut à pluribus legeretur.* Sed quibus *principia* poſita *ſatìs intellecta* non fuerint, ii *vim conſequentiarum* minimè percipient, neque *præjudicia* deponent, quibus à multis retrò annis inſueverunt : Et propterea, ne res in *diſputationes* trahatur, ſummam libri illius tranſtuli in Propoſitiones, more mathematico, ut ab iis ſolis legantur qui *principia* priùs evolverint." *Præfat.* in *Lib.* III. *Princip.*

6. Nec extant testimonia, *Newtonum* methodum suam fluxionum cuivis communicâsse ante annum 1672; regerit enim Newtonus, in brevi illâ historiâ de fluxionum inventione, *Schol. Lem.* II. *Lib.* II. *Princip.* 3tiæ edit. 1726, p. 246, in his verbis.

"In epistolâ quâdam ad *D. Collinsium* nostratem 10 Decemb. 1672 datâ, cum descripsissem *methodum Tangentium* quam suspicabar eandem esse cum methodo *Slusii* (tum nondum communicatâ), subjunxi:"

—'*Hoc est unum particulare, vel* COROLLARIUM *potiùs,* METHODI GENERALIS *quæ extendit se citrà molestum ullum calculum, non modò* AD DUCENDAS TANGENTES *ad quasvis curvas, sive geometricas sive mechanicas; vel quomodocunque rectas lineas, aliasve curvas, respicientes; verùm etiàm ad resolvendum alia abstrusiora problematum genera* DE CURVITATIBUS, AREIS, LONGITUDINIBUS, CENTRIS GRAVITATIS CURVARUM, &c. *Neque (quemadmodum* HUDDENII *methodus de* MAXIMIS *et* MINIMIS) *ad solas restringitur æquationes quæ quantitatibus surdis sunt immunes.* HANC METHODUM *intertexui alteri isti* [DE ANALYSI PER ÆQUATIONES NUMERO TERMINORUM INFINITAS, *quam anno* 1671 *de his rebus scripseram*] *quâ æquationum exegesin instituo reducendo eas ad series infinitas.*"—

7. Alteram quoque epistolam, 13 Jun. 1676, scripsit *Newtonus* ad *Oldenburgum*, cum *Leibnitzio* communicandam, in quâ celeberrimum *Theorema Binomium*, primò, absque demonstratione proposuit, (nec unquam posteà demonstratione ipse munivit) cujus ope nimirùm, "*æquationum illam exegesin*" potissimùm, instituerat. Hoc autem, ex intimis penetralibus fluxionum haustum, *Newtono*, saltèm ante *Pestem Londinensem*, anno 1665, innotuisse, anteà memoravimus, ANALYS. ÆQUAT. p. 33. not. Vide quoque § 96 hujus.—Cæterùm hæ duæ epistolæ, *Leibnitzianam* illam Jan. 21, 1677, tempore antegressæ, *primi inventoris* honorem *Newtono* servare videntur; nec inficiabitur æqua posteritas.

8. De famâ *Leibnitzii* plus æquo forsàn detrectans, cœtus Analystarum *Anglicorum* adjudicavit, "*That in the letter of* 10*th December* 1672, from *Newton* to *Collins*, the method of fluxions was *sufficiently* described to any *intelligent* person."—Verùm, si "hæc methodus *satìs* in istâ epistolâ *descripta fuit ut ab aliquo harum rerum modicè perito posset intelligi*, quæri potest," cur *Newtonus*, anno 1677, ad æmulum suum "*intelligentissimum* certè atque *peritissimum*" *Leibnitzium* scribens, anagrammate quodam *methodum suam fluxionum directam* diutiùs celaret?—Disputationes sanè quæ adhuc, etiàm nostro ævo, vigent de evidentiâ et certitudine hujus methodi, inter Analystas perspicacissimos, scholæ tam *Newtonianæ* quam *Leibnitzianæ*, (non obstantibus, vindiciis, expositionibus, et commentariis discipulorum *Newtoni* celeberrimorum) satìs ostendunt cur *tam tardè eruantur, quæ tam altè jacent.*—*Seneca.*

Adde quod, si verum fuisset, quod ex scriptis *Newtonianis* primos theoriæ suæ igniculos elicuerat *Leibnitzius*, haud mediocris laus idcircò illi concedenda videretur, qui arcanum molestissimum et longè difficillimum in lucem ex tenebris proferre poterat:

Quem vituperare ne inimici quidem possunt, nisi ut simùl laudent.

9. Veruntamen tutò colligere licet, neque *Newtonum* neque *Leibnitzium* pro inventore absoluto hujúsce methodi habendum esse; nam si manifestò constat,

utrumque methodum suam à methodo *ducendi Tangentes Barrovianâ* mutuâsse; nonne utrique laudem *pleni perfectique ab integro inventoris*, detrahet æquus controversiæ judex?

Audiamus *Leibnitzium* in epistolâ quâdam April 9, 1716, datâ:

"M. *Newton* hazarde icy un accusation qui va tomber sur lui-même: Il prétend que ce que j'ay écrit pour lui à M. *Oldenbourg* en 1677 *est un déguisement de la méthode de M. Barrow.* Mais, comme M. *Newton* avoue dans la page 253 et 254 de la première édition de ses principes:"—'*Me, [Leibnitzium] ipsi [Newtono] tunc methodum communicâsse à methodo ipsius vix abludentem præterquam in verborum et notarum formulis*'—" il s'ensuivra que *sa méthode aussi n'est qu'un déguisement de celle de M. Barrow.*"—Et non infelicitèr sanè retorquetur à *Leibnitzio*; *Newtonum* enim potiùs mutuâsse ab amico, socio, studiorúmque patrono, ipsiúsque prædecessore in munere professorio *Lucasiano* CANTABRIGIÆ, optimo eruditissimóque *Barrow*; quàm *Leibnitzium* suam methodum ab eodem Barrow derivâsse, scriptore, scilicet, quoad Leibnitzium peregrino et fortasse parùm noto, longè verisimilius est.

10. Quantum verò debuit *Newtonus* antecessoribus suis, ipse haud rarò ingenuè fatetur—modestiâ quàm ingenio non minùs spectabilis. Debuit *Slusio* quidem in *methodo Tangentium*, imprimísque *Barrow*, in methodo ducendi Tangentes per *triangulum arithmeticum*; *Nicolao Mercatori*, in *Logarithmotechniâ*, quam à *divisione fractionum* ad *extractionem radicum* per *theorema binomium*, extendit Newtonus; *Hugenio*, "in eximio suo tractatu *de horologio oscillatorio*," cujus methodum determinandi *vires centripetas et centrifugas* corporum revolventium in *circulis*, ad vires corporum revolventium in *curvis quibuscunque*, solertissimè extendit Newtonus per *circulos ejusdem curvaturæ*; debuit etiàm *Wallisio*, in *quadraturâ curvarum*, reducendo *ordinatas irrationales* ad *rationales*, per *series infinitas*; *Fermatio*, in solutione problematum de *maximis et minimis*; *Cavallerio*, in *methodo indivisibilium*; atque ipsi *Cartesio*, in applicatione *Algebræ ad Geometriam*, quam ipse tantoperè auxit, atque ad *mechanicam rationalem*, (seu "*scientiam motuum* qui ex *viribus quibuscunque* resultant, et *virium* quæ ad *motus quoscunque* requiruntur, accuratè propositam ac demonstratam")—solertissimè extendit et accommodavit.

11. *Magnus* sanè habendus est ille *inventor*, qui obscura et involuta scientiæ arcana, arte suâ indagatrice, enucleavit et detexit, lucem ex tenebris promens; *major* tamen esse videbitur, si *ipse expositor*, quæ detexit, ea *noverit* et *voluerit* certâ ac idoneâ ratione demonstrare;—"*analysi brevissimâ et simul perspicuâ, synthesi concinnâ et minimè operosâ*," qualem exercebat olìm magnus ille Geometra *Apollonius*, in libro suo eximio de *Sectione Rationis*, testante *Halley*; atque minimè deficit ab ejus vestigiis *Newtonus*, in *Lemmate* II. *Lib*. II. *Princip*. imperatoriâ quâdam brevitate, et lucido ordine, fundamentum methodi suæ generalis fluxionum positurus.

12. Veruntamen fatendum est, quod *Newtonus*, nimìs forsàn brevitatis studiosus et moram et molestiam interpretandi refugiens, non nisi "*peritos alloquens*," semina tantùm expositionis veræ atque genuinæ sevit, à posteris serò explicanda in frugem.

frugem uberiorem justæ atque legitimæ demonstrationis, ubi "*Opinionum commenta deleverit dies;*" et tandem, post lapsum annorum centùm, consensu communi sectatorum scholæ *Newtonianæ* et *Leibnitzianæ*, methodi ipsius generalis sive *fluxionalis* sive *differentialis*, rudimenta meliùs intelligantur, ejúsque elementa prima "*quæ tam altè jacent,*" imis in penetralibus scientiæ *metaphysicæ*, cautiùs atque curatiùs "*eruere,*" et argumentis idoneis et ad naturam scientiæ illustrandæ accommodatis tandem confirmare, fas sit.

13. Neutiquam cuivis homini contingit inventa sua, à primordiis ad ultimum perfectionis statum, spatio vitæ vel longissimæ perducere. Artis scientiaéve cujusvis elementa simùl invenire, et inventa demonstrare, vel maximis ingeniis rarissimè concessit Deus. Gradatìm et seriatìm fit VERITATIS exploratio.—"*Plantabat Paulus, irrigabat Apollos, sed* DEUS *augebat*"—haud minùs verum est in *Philosophiâ* quam in *Religione.*

"*Felix*" quidem censendus est, "*qui potuit*" vel *paucos* annulos arripere tantùm, eosdémque *infimos*, immensæ illius catenæ VERITATUM ÆTERNARUM, quam à cœlis ad tellurem usque demisit, ægris mortalibus et hallucinantibus, benignus ILLE DEUS, qui variis gaudet nominibus—῾Ο ΛΟΓΟΣ ΤΟΥ ΘΕΟΥ, ῾Η ΣΟΦΙΑ, seu "RECTA RATIO," "SAPIENTIA," "VERITAS,"—tàm apud philosophos profanos quàm apud sacros; et qui hanc catenam non viris ingenii et scientiæ suæ opinione tumidis, sed potiùs iis qui humiliùs de se sentiunt et vitæ et morum sanctitate insignes fiunt, porrigit,—*paucis* quidem "*quos æquus amavit* JUPITER"—"*Divûm hominúmque æterna potestas*; tales versùs sese, lenitèr et indesinentèr attrahens atque attollens:—alligatur enim *summus* hujúsce catenæ annulus ad basin ipsius *throni divini.* Vid. *John* xii. 32.

Και εδικαιωθη ῾Η ΣΟΦΙΑ απο των τεκνων αὐτης ἁπαντων. *Luk.* vii. 35.

Licèt enim pusilli *Sophistæ*, seu *Pseudo-Philosophi*, *veritates æternas* haud rarò obfuscent, ipsámque "ALMAM SAPIENTIAM" vilipendant et obtrectent ingrati; "*indies nihilominùs,*" animosè vindicatur et "*justificatur, liberis suis omnibus*" verè philosophantibus, à primordiis rerum; quales identidem prodierunt, "*lucida sidera,*" præclara lumina mundi: (*b*) ENOCH, NOAH, JOB, ABRAHAM, MOSES, DANIEL *Archimagus*, et PAULUS *Tarsensis*; THALES, PYTHAGORAS, SOCRATES,

(*b*) 1. ENOCH antediluvianus, et "ab *Adamo* septimus," transcendentem ob pietatem vivus translatus est in cœlum, ætatis suæ anno 365mo; veros dies anni *solaris* mirè exponente; DEO forsàn, illi, quasi *astronomo*, id revelante; sicut illi, quasi *prophetæ*, revelavit *statum futurum*, et *judicium generale*, quæ prædixit *Enoch*, denunciavítque impiis, *Jude* xiv.—Hoc enim (licèt inficiantur *semi-philosophi* et *semi-theologi* hodierni) dogma fuit SAPIENTIÆ PRIMÆVÆ antiquissimum:

῾Οτι ῾Ο ΘΕΟΣ ΕΣΤΙ, και τοις εκζητεσιν
Αυτον ΜΙΣΘΑΠΟΔΟΤΗΣ ΓΙΝΕΤΑΙ.

"*Quod* DEUS EST, et exquirentibus Ipsum
REMUNERATOR fit." *Heb.* xi. 6.

Et "*ultimis* quoque *diebus,*" τῃ παλιγγενεσιᾳ,—ubi "*magnus ab integro sæclorum nascitur ordo*"—"*dicet vir*" omnis verè philosophans:

SOCRATES, PLATO, ARCHIMEDES, et HIPPARCHUS ὁ φιλαληθεστατος; COPERNICUS, GALILEO, ROGERUS BACON et FRANCISCUS BACON, PASCAL, BOYLE, BARROW, NEWTON, MACLAURIN, COTES, DANIEL BERNOULLI, &c. &c. &c. et noviſſimè quidem doctiſſimus vir, Gulielmus Jones, eques auratus, Curiæ Supremæ Britannicæ apud Calcuttam in Indiâ Juſticiarius, (Gulielmi Jones, celeberrimi Mathematici ſub initio hujus ſæculi, filius,) *Indorum Apoſtolus*: qui, ſuis in ſæculis non magìs claruerunt omnes, famâ ſcientiæ reconditioris, quam virtutis pietatíſque ſublimioris; et etiamnum, unuſquiſque eorum *αποθανων ετι λαλειται*, "*licet mortuus, adhuc laudantium in ore eſt*;" honóſque eorum in æternum vigebit.

De Objectionibus allatis contrà Methodum Fluxionum à præcipuis Auctoribus qui hactenùs impugnant ejuſdem Evidentiam et Certitudinem.

---

14. CONFESTIM à primâ expoſitione perbrevi *Methodi Fluxionum*, novæ, ſubtilioris et reconditioris, objectiones variæ ſuboriebantur, Analyſtarum tam peritorum

---

"*Certè fructus eſt juſto*:
Certè DEUS, *judicans in terrâ*." *Pſal.* lviii. 11.

"*An* JUDEX TOTIUS TERRÆ *non faciet judicium?*" *Gen.* xviii. 25.

2. NOAH, magnus ille propheta et inſtaurator generis humani; "*qui gratiam invenit apud* DEUM," *à diluvio* ſervatus; quod prædixit, fruſtráque denunciavit, 2 *Pet.* ii. 5. et *fortunas Noachidarum* cecinit moribundus, *Gen.* ix. 25.—Et "Inſtitutionis *Menu*" apud *Indos*, (qui *Manes* eſt dictus apud Ægyptios,) etiamnum traditione celebrantur.—*Cyclus magnus* 600 annorum, Antidiluvianus, à *Joſepho* notatus, ætatem *Noæ* tempore diluvii mirè ſervat.

3. *Job*, à Diluvio ſeptimus (Vide *Abulfaragi*, p. 13.)—*ὁ μεγας οντως εκεινος και γενναιος της αληθειας αγωνιστης*—"*magnus ille et nobilis* VERITATIS *vindex*"—*Suidas*—peritiſſimúſque *aſtronomus*, qui conſtellationes cardinales ſphæræ primitivæ recenſet, *Job* ix. 9. et xxxviii. 32. AISH, CHIMAH, CHESIL et MAZAROTH, ſive "*Urſa Major*, *Taurus*, *Scorpio* et *Canis*" reſpectivè; ut videre licet in opere quodam an. 1799 edito, cui titulus, THE INSPECTOR—*London*, *White*.

4. ABRAHAM,—"*Amicus ille* DEI," et "*Pater fidelium*;" idémque peritiſſimus aſtronomus; ſicut per totam Orientem adhuc celebratur:—*Ασκηθεις, κατα τον πατριον νομον, τας των επεξανιων αστερων κινησεις· και στοχασαμενος ως ουκ εν τουτοις ἱσταται το μεγαλουργον της φαινομενης ταυτησι κτισεως, αλλ' εχει τινα τον* ΔΗΜΙΟΥΡΓΟΝ *και κινουντα και διευθυνοντα την εναρμονιον των αστερων πορειαν, και του κοσμου παντος την καταστασιν.*

"Exercitatus, patrio more *Chaldæorum*, in *aſtrorum motibus* obſervandis: atque conjecturâ aſſecutus eſt, quod non in *iis ipſis* conſiſtit *vis magna effectrix* apparentis hujúſce creationis; ſed quod potiùs habet aliquem OPIFICEM PUBLICUM, qui et movet et dirigit *harmonicum* aſtrorum curſum et mundi univerſi conſtitutionem."——*Suidas*.

5. MOSES,—maximus ille legiſlator, "*in omni ſapientiâ* ÆGYPTIORUM *inſtructus*;" qui inſtitutione *Paſchæ*, (Deo forsàn revelante,) cyclum illum *decennovalem lunæ*, celeberrimum tradidit, in computandis noviluniis, et *paſchalibus* pleniluniis, apprimè neceſſarium; cujus ope, *Meton*, longo poſt tempore, Calendarium *Athenienſe* emendavit.

6. DANIEL.

peritorum quàm imperitorum, validitatem ejus denegantium; ac, (non obſtantibus magnis illis Newtoni diſcipulis,—COLSON, qui anno 1736, *Newtoni Methodum Fluxionum Analyſi Æquationum per ſeries infinitas intertextam*, verſione Anglicâ et commentario copioſo donavit et illuſtravit;—MACLAURIN, qui anno 1742, *Tractatum* ſuum *Fluxionum*, Anglicè edidit, fundamentum totius methodi generalis à primordiis ad mentem *Newtoni* in *Lemmate Secundo*, Lib. II. Princip. explicare profeſſus;—et STEWART, qui anno 1745, duo ipſius Newtoni celeberrima opera, *Analyſin Æquationum per ſeries numero terminorum infinitas*, et *Quadraturam Curvarum*, verſione Anglicâ et commentario uberrimo inſtruxit;—aliiſque ſcholæ *Newtonianæ* diſcipulis celeberrimis, *Simpſon*, *Saunderſon*, *Emerſon*, *Waring*, &c.) adhuc vigent objectiones tàm noſtratium *Berkeley*, *Landen*, &c. ac nuperrimè ſcriptoris doctiſſimi in libro cenſorio, ſingulis menſibus edito, cui titulus, *The Monthly Review*; quàm exterorum *D'Alembert*, *Torelli*, *La Grange*, &c. optimorum, ſcilicet, expoſitorum *Methodi Differentialis*, et acerrimorum obtrectatorum *Methodi Fluxionum*.

Et certè fatendum eſt, non obſtantibus tot commentariis tantorum Analyſtarum, qui fines Fluxionum magnoperè extenderunt, hactenùs deſiderari ipſius *Textûs Newtoniani*, (breviſſimi ac reconditiſſimi,) interpretem fidum atque perſpicuum, methodi et lucidi ordinis ab ipſo auctore inſtituti ſtrenuè ſequacem, et expoſitione nec nimis curtâ, nec nimis prolixâ, difficilia illuſtrantem; (vitia, quæ utcunque ſibi mutuò contraria videantur, tamen haud rarò ſimùl et ſemèl inveniuntur;) ac de ſenſu nativo ipſius authoris eruendo, quàm de ſuâ eruditione aut facundiâ oſtentandâ, magìs ſollicitum.

6. DANIEL ille *Archimagus*, in omni ſapientiâ et diſciplinâ *Magorum* et *Chaldæorum* inſtructus, atque ita celebratus ut "*ſapientior Daniele*" in proverbium eſt evectum, per totam Orientem, *Ezek.* xxviii. 3.—Et in Sacris Scripturis honoratiſſimè diſtinguuntur—"*Noah*, *Daniel* et *Job*"—quaſi præpotentes *interceſſores* cum DEO—*Ezek.* xiv. 14.—*Noah*, pro ſe ſuâque familiâ, liberatis ab excidio diluvii—*Daniel*, pro reditu *Judæorum* Captivorum;—et *Job*, pro amicis ſuis temerè philoſophantibus, et PROVIDENTIAM DIVINAM ignavè vindicantibus.

7. PAULUS *Tarſenſis*—"apud pedes *Gamalielis*, celeberrimi præceptoris *Judæorum*, educatus;" *Acts* xxii. 3. et in omni diſciplinâ ſcholarum philoſophicarum *Alexandriæ*, *Athenis* et *Romæ*, non levitèr inſtructus.

8. THALES, eclipſin illam ſolarem prædixit, quæ diremit pugnam inter *Lydios Medóſque* commiſſam, *Julii* 30, *anno* 607, *ante æram Chriſti vulgarem*, (ut poſthac forſàn probare mihi dabitur occaſio in *diſſertatione* quâdam *De Chronologiâ Antiquâ*, mox edendâ—ſi DEUS *det vitam*, *det opes*)—epochâ, nimirùm, ab aſtronomis chronologíſve hactenùs non ſatìs notatâ. Hanc verò eclipſin, tantoperè litigatam, facilè computare potuiſſet *Thales*, ope *Cycli* illius *Decennovalis*, ex *Judæis Chaldæiſve* mutuati—à *Græcis* et *Metone*—qui accuratiùs conſtabat ex *lunationibus* 223, ſpatio *annorum* 18 *cum diebus* 11, peractis—quem figuratè et aptè innuit *Theocritus*, *Idyll.* XV.

Οκτωκαιδεκαίης η εννεακαιδεκ' ὁ γαμβρὸς——

De nuptiis Adonidis loquens, ſeu *conjunctionibus Solis et Lunæ*—"*octodecimo vel nonodecimo anno.*"

9. PYTHAGORAS, ille *Samius*, cujus allegoriam eximiam de ſemitis vitæ divergentibus ad *veritatem* vel ad *errorem*, per furculas literæ Υ, deſignatis, notat *Perſius*, *Sat.* III. 56.

Et tibi quæ SAMIOS *deduxit litera ramos,*
Surgentem DEXTRO *monſtravit limite callem.*

Quam lucubentiſſimè exponere non dedignatus eſt ipſe JESUS CHRISTUS, *Matt.* vii. 13.—Verum quoque mundi ſyſtema, ſeu *Copernicanum*, aut detexiſſe, aut ſaltèm ex *Chaldæis* didiciſſe, videtur.—Vide *Maclaurin's Account*, *Chap.* 2.

15. Audiamus THE MONTHLY REVIEW, in eruditâ admodùm recenſione ſuâ libri cui titulus *Theorie des Fonctions Analytiques*, par *La Grange*, *Paris* 1798. *Art.* 1. *Append.* Vol. 28.

" *The Method of Fluxions* had never ſo acute, ſo learned, and ſo judicious a defender as *Maclaurin*.—His work is moſt valuable for the variety of information which it contains, and for many ſpecies of clear and forcible argumentation: yet whoever conſults it, and ſees the various *artifices* which he is obliged to employ for the purpoſe of demonſtrating the ſeveral parts of the method; and the grounds of it not eſtabliſhed in leſs than *fifteen long theorems*; and, in an enquiry *purely mathematical*, finds the author ſpeaking of "cauſes and effects," "of the ſprings and principles of things," and propoſing to deduce "the relations of quantities by comparing the powers which are conceived to generate them;"—will be convinced that this could only happen from ſo able a mathematician having failed to ſeize *the right principle* and the *true mode of explanation*."

16. Atque hujus acutiſſimi cenſoris ſententiæ multis nominibus accedo: nam, primò, à verâ interpretandi viâ deflectens, *Maclaurinus* illuſtrationibus geometricis, à naturâ *motûs* petitis, nimìs immoratur in libro primo; longo ſermone luxurians, et argumentationis ambagibus ad faſtidium uſque indulgens, à genio *Newtoniano* prorsùs alienis; qui *methodum ſuam Fluxionum* excogitavit, " *ut effugeret tædium deducendi longas demonſtrationes, more veterum geometrarum, ad abſurdum*;"—et " *contractiores* " his, et ſimùl accuratiores illis quæ per *methodum indiviſibilium* à Cavallerio ejúſque diſcipulis eliciebantùr, magno ſcientiarum emolumento, ſubſtituit. Contrà, dum brevis eſſe laborat, obſcurus fit *Maclaurinus* in libro ſecundo; et, quaſi interpretandi pertæſus, primo hujus capite, ſtrictìm tantùm, principiis veris ac genuinis methodi univerſalis, (ex naturâ *relationum* quantitatum variabilium abſtractè conſideratarum, manantibus,) vix, aut levitèr tantùm notatis, *elementa prima* ſeu *metaphyſica*, à *Newtono* poſita, feſtinantèr tranſcurrit; et *principia mathematica* quoque nimiâ et præproperâ celeritate abſolvit; et denique, ipſius inventoris lucidum ordinem ſtatìm ab initio deferens, multa mutat; et quæ mutat, corrumpit: nam,

2. *Prop.* 1. Totius theoriæ ſuæ fundamentalem, § 707, " *fluxionem quadrati* A^2, *æquari* $2aA$;" non niſi imperfectè demonſtrat; unde enim hanc fluxionem *aſſumpſerat* (*c*), minimè oſtendens, tranſit, per ſaltum, ad demonſtrandam veram fluxionem hâc, nec *majorem* eſſe, nec *minorem*, argumento ad abſurdum prolixiore uſus.

(*c*) The defect of *Maclaurin*'s demonſtration may thus be ſupplied, from his own principles rightly conceived, though rather confuſedly expreſſed, § 701—706.

§ 707. *Prop.* 1. *The fluxion of the root* A *being ſuppoſed equal to* a, *the fluxion of the ſquare* AA *will be equal to* 2aA.

Let $A - a$, A, and $A + a$, be ſucceſſive values of the variable root; and the correſponding values of the variable ſquare will be $A^2 - 2aA + a^2$, A^2, and $A^2 + 2aA + a^2$; of which the firſt is

ufus. Et inde, § 708, "*fluxionem rectanguli* A + B *æquari* $aB + bA$," pro corollario, deducit. At artifex ipſe, longè elegantiùs atque peritiùs, auſpicando à *caſu primo*, et generaliore,—*fluxione* nempe *rectanguli*, inde particularem,—*fluxionem* nempe *quadrati*,—facillimè deducit, *Caſ.* 3.

3. Per varios anfractus procedens, *lemmata* compluria, ad demonſtrandas propoſitiones inſequentes, tortuosè introducit *Maclaurinus*, à quibus immunis erat *Methodus Newtoniana*.

4. Ad methodum inventoris jam ſerò reverſus, quam adeò infelicitèr initio deſeruerat, breviùs et meliùs demonſtrat "*fluxionem* A^n *æquari* naA^{n-1}," more *Newtoniano*, § 716, quam imperfectè et prolixè demonſtraverat anteà, § 712, ſecutus *Prop.* 1.

5. Negligentiâ minimè condonandâ, *Theorematis illius generaliſſimi, Lem.* II. *Lib.* II. *Princip. ultimum caſum*, quem à præcedentibus, directo atque immediato nexu, ſummo acumine, ſummâque peritiâ Algebraicâ, deduxerat *Newtonus*, nempe, *fluxionem* A^mB^n *æquari* $maB^nA^{m-1} + nbA^mB^{n-1}$, prorſùs omittit; itémque *tria corollaria* inſequentia; elegantiſſima atque uberrima quidem, quæ calculo fluxionali concinnando, ubi de *proportionalibus* agitur, et de *logarithmis* præſertìm, magnoperè inſerviunt, eâdem negligentiâ omiſit.

is the leaſt, and the laſt the greateſt. From this laſt value of the ſquare ſubtract the firſt, and, the extreme terms A^2 and a^2 being deſtroyed, there will remain the *exceſs* $+ 4aA$; which will denote the *whole* increaſe of the ſquare from the firſt $(A - a)^2$ to the laſt $(A+a)^2$, with the increment a, *twice*, or $2a$; (which is the difference between the extreme values of the root, $A-a$ and $A+a$); and conſequently *half the exceſs*, namely $2aA$, will denote the increaſe with the increment a *once*; or will *meaſure the rate of variation*, or be *the fluxion* of the variable ſquare, from its *leaſt* to its *mean* ſtate AA.

If you deny it, let the true fluxion be either greater or leſs than $2aA$, by any aſſignable difference D, or equal to $2aA \pm D$. But if greater, the laſt increment muſt neceſſarily be greater than $+ 2aA + a^2$; if leſs, the firſt increment muſt be leſs than $+ 2aA - a^2$ (or the difference between the firſt and mean values of the variable ſquare), contrary to the hypotheſis. Q. E. D.

In numbers the caſe is plain:

Let *the root* A = 10; and its *momentary increment* $a = 1$.

	Val.	*Diff.*
$(A - a) \times (A - a)$ =	$9 \times 9 = 81$.	19
$A \times A$ - - - - - - - =	$10 \times 10 = 100$	
$(A + a) \times (A + a)$ =	$11 \times 11 = 121$	21 / 40.

$$\text{But} \; - \; - \; 4aA \; - - - = 4 \times 10 = (40 =) \; 19 + 21.$$

$$\text{Therefore } 2aA \; - - - = 2 \times 10 = (20 =) \; \frac{19 + 21}{2}.$$

Conſequently $2aA$ expreſſes the *fluxion* or *rate* of increaſe of the variable ſquare in its advance to the *mean* ſtate A × A.—The demonſtration might be abridged upon *Newton*'s principles, by taking only the half increment:—For the leaſt value $(A - \frac{1}{2}a)^2$ becomes $A^2 - aA + \frac{1}{4}a^2$, and the greateſt, $(A + \frac{1}{2}a)^2 = A^2 + aA + \frac{1}{4}a^2$. But the exceſs, $+ 2aA$, gives *the fluxion* at once, during the *whole* momentary increaſe. See § 84 and 85.

17. Diluendis igitùr objectionibus eorum qui methodi fluxionum generalis evidentiam atque certitudinem impugnant, omninò imparem esse commentarium *Maclaurini*, quamvis valdè eruditi, et in hisce scientiis versatissimi, et (quod auctoritatem majorem ipsi parere debere super hoc argumento videatur,) in ipsius *Newtoni* amicitiam (*d*) admissi, quis mathematicus non videt?—At non levi detrimento theoriæ fluxionum, iste commentarius fusior, qualis qualis fuerit, mox supplantavit originalem textum, paullò subtiliorem atque difficiliorem intellectu, ob *imperatoriam* auctoris *brevitatem*, non nisi auribus rite purgatis capiendam—

Φωναντα συνετοισι.

Ex hoc fonte igitùr profluxerunt vel perspicaciorum peritiorúmque analystarum objectiones, scholæ *Leibnitzianæ* præsertìm, adversùs ipsam fluxionum methodum; quasi methodo *differentiali*, evidentiâ et certitudine, cedentem, et in principiis à geometriâ alienis fundatam.

18. Audiamus *D'Alembert*, analystam adeò celebratum, in *Mathesi* perspicacissimum, *in religione* hebetissimum, argumento paulò subtiliore validitatem exponendi *fluxiones* per velocitates, impugnantem:

" Introduire ici *le mouvement*, c'est y introduire une idée *étrangère*, et qui n'est point *nécessaire* à la démonstration : d'ailleurs, on n'a pas d'idée bien nette de ce que c'est *la vîtesse* d'un corps à chaque instant, lorsque cette vîtesse est variable. La vîtesse n'est rien de *réel*, c'est le *rapport* de l'*espace* au *tems*, lorsque la vîtesse est *uniforme*; mais lorsque le mouvement est variable, ce n'est plus le rapport de l'espace au tems, c'est le rapport de *la differentielle* de l'espace à cette

(*d*) The following letters of *Newton* reflect high honour on his liberality of mind and delicacy of friendship towards the excellent *Maclaurin*, when a candidate for the joint *Professorship of Mathematics* in the University of *Edinburgh*.—In one of these letters, addressed to Mr. *Maclaurin*, (with permission to shew it to the Patrons of the University,) Sir *Isaac* expresses himself thus:

" I am very glad to hear that you have a prospect of being joined to Mr. *James Gregory* in the Professorship of the Mathematics at *Edinburgh*, not only *because you are my friend*, but principally *because of your abilities*; you being acquainted as well with the *new improvements* of Mathematics, as with the *former state* of those sciences: I heartily wish you good success, and shall be very glad of hearing of your being elected. I am, with all sincerity, your faithful friend and most humble servant."

In a second letter, which was addressed to the then Lord Provost of *Edinburgh*, (which Mr. *Maclaurin* knew nothing of till some years after Sir *Isaac*'s death,) he thus writes:

" I am glad to understand, that Mr. *Maclaurin* is in good repute amongst you for his skill in Mathematics; for *I think he deserves it very well:* And, to satisfy you that I do not flatter him, and also to encourage him to accept the place of assisting Mr. *Gregory*, in order to succeed him, I am ready (if you please to give me leave,) to contribute *twenty pounds per annum* towards a provision for him, till Mr. Gregory's place becomes vacant, if I live so long, and I will pay it to his order in *London*."

See an *Account of the Life and Writings of Maclaurin*, prefixed to his *Account of Sir Isaac Newton's Philosophical Discoveries*, in which are many curious and valuable particulars of the life of that Christian Philosopher, especially in his last moments; so different from those of that *miserable* infidel *D'Alembert*.——Appendix II. § 161 & 164.

du

du tems; rapport dont on ne peut donner d'idée nette que par celle des *limites*. Ainsi *il faut nécessairement en revenir à cette dernière idée, pour donner une idée nette des fluxions*."

19. Quinquaginta retrò annis circitèr, talia objecit vir clarissimus *D'Alembert*, cujus auctoritas nunc tanti æstimatur. Ac nuperrimè quidem, ("*vires* enim *acquirunt eundo*,") maximus ille promotor Algebræ puræ, nec non analyseos sublimioris, *De La Grange*, in opere quodam elementari, cui titulus, THEORIE DES FONCTIONS ANALYTIQUES, 1798, *Paris*, autumat se serò tandem vera et genuina principia *calculi differentialis* extudisse, ab omni consideratione sive *quantitatum infinitè parvarum*, seu *evanescentium*, sive *limitum* vel *fluxionum*, libera atque immunia; usurpando tantùm *Quantitatum finitarum Analysin Algebräicam*.

20. Hísce magni nominis viris consentiens *Landen* nostras, ita doctrinam fluxionum perstringit in *Analysi suâ residuâ*, anno 1758—Analysta et ipse ex peritissimis.

"It seems more proper, in the investigation of propositions by *Algebra*, to proceed upon the *anciently-received principles* of that art, than to introduce therein, *without any necessity*, the *new* fluxionary principles derived from a consideration of *motion*: and the rather, as the introduction of these new principles is not attended with any *peculiar* advantage."

21. Tantis opponentibus accedit quartus,—nimirùm *censor* ille, in hísce scientiis non levitèr versatus, (quisquis fuerit,) apud THE MONTHLY REVIEW, *Append.* Vol. 28, 1799,—ex cathedrâ moderatoris, ita pronuncians judicium, in *Newtonum* ejúsque commentatores et discipulos:

"Of the *Method of Fluxions*, properly so called, *Newton* is the sole and undisputed inventor; to him, beyond all doubt and controversy, is to be attributed the *establishment* of that doctrine on the consideration of *Motion*.

"Had not *Newton* been the inventor of *this* method of considering Fluxions, WE are of opinion, that very few persons would have contended either for its *metaphysical* or *mathematical* excellence: for, on the ground of perspicuity and evidence, the understanding is not much assisted by being directed to consider *all* quantities as generated by *Motion*. *Lines* we can conceive generated by the motion of *points*; *surfaces*, by the motion of *lines*; &c. But when such quantities as *Weight*, *Density*, *Force*, *Resistance*, &c. become the objects of enquiry, and are said to have their *flux* and *velocity*, then the true end of the *figurative* mode of speech—*Illustration*—is lost: On the ground of *mathematical* convenience—that the definition of Fluxions, as given, is commodious to deduce thence their properties and affections—nothing is gained; since the enquiry is reduced "*ultimately*" (*en dernière analyse*) to the determination of *the ultimate ratio* of vanishing quantities, or of *the limits* of such quantities: which latter method is only the former, *algebräically* expressed.

 "That

"That which happened to *Ariſtotle* has happened to *Newton*: His followers have bowed ſo implicitly to his *authority*, that they have not exerciſed their *reaſon*.—If the *Engliſh* Mathematicians firſt adopted *Newton*'s Method from veneration to him, or *for want of better information*, they have ſince perſevered in it, (WE may almoſt ſay) *againſt conviction*.—

"It has been ſaid, that *the true principles of any invention are moſt ſatisfactorily and truly ſtated by the inventors themſelves*;—yet it is at the diſtance of more than a hundred years from the diſcovery of the *Fluxionary Calculus*, that its principles are firſt clearly and rigorouſly eſtabliſhed. It has been reſerved for the Mathematicians of *theſe days*, to effect what was denied to a *Newton* and a *Leibnitz*; and to ſupply to their theory that evidence and exactneſs, the want of which makes us attribute its invention rather to the felicity of the times in which they lived, than to the excellence of their genius:—to eſteem it, not *pro partu ingenii*, but *pro partu temporis*."

---

22. Eruditorum ſanè graves ſunt hæ objectiones, et gravitèr pronunciatæ. Veruntamen, non obſtantibus tantis judicibus, et poſthabitâ omni *auctoritate*, (quæ in MATHESI minimè valere debet,) fidentèr affirmare auſim, propriis ex inveſtigationibus, *Nullius addictus jurare in verba magiſtri*—à nemine, ne vel ab ipſo *De La Grange*,—evidentiùs, accuratiùs, ſolidiùs, invictiùs et generaliùs, ſtrui fundamentum totius *Theoriæ Fluxionum*, quàm à celeberrimo inventore, *ſagaciſſimi ingenii* viro; ſi modò, miſſis aliorum commentis, adeamus ipſius *textum originalem*; nam, ſi attentè et circumſpectè hic textus inſpiciatur, et, contextu examinato, cæteríſque auctoris documentis criticè collatis, probè intelligatur, terminis technicis aptè definitis, atque ellipſibus ritè ſuppletis, (quæ cauſæ ſunt ex quibus præcipuæ difficultates in Newtoni ſcriptis oriuntur) cenſebit, nì fallor, omnis æquus peritúſque controverſiæ hujus judex, objectiones hactenùs prolatas, vel graviſſimas, ex inſcitiâ potiùs, ex imperitiâ, præjudicio, vel livore, quàm ex juſtis ratiociniis et veritatis perceptione ortas eſſe; imò in ipſos objectores jure retorqueri poſſe.

23. Analyſtas *exteros* quidem, eoſdémque *Gallos*, *nomini Britannico* olìm infenſos et nunc acerbiſſimè,—*D'Alembert* aut *De La Grange* (*e*)—ipſius *Newtoni* æmulos minimè impares, de laude ejus detrectare voluiſſe, non eſt mirum; qui nec

(*e*) DE LA GRANGE.—Quindecim retrò annis, hæc notabam fidus NEWTONI diſcipulus. ANALYS. ÆQUAT. § 259.

"Ex dictis facilè judicabit lector, an valent multi nominis viri *La Grange* objectiones. *Diſſert. Acad. Berlin*, an. 1768: nempè quod methodo *Newtonianâ* [extrahendi radices æquationum quàm proximè, in *Analyſi Æquationum per ſeries infinitas*, quàm demonſtratione generali munieram § 249—259.] ignoratur quantum ad veritatem accedit tùm valor (k) primò aſſumptus, tùm valor cujuſque complementi inſequentis (p), (q), &c.—Methodus certè quam ut accuratiorem commendat, *Newtonianæ* facilitate longè cedit; quippe quæ ex inventione quantitatis $\sqrt{l}$, § 234, non niſi per *tentamina*,

nec rarò nec parcè, quamvis furtìm, inventiones ejus reconditissimas enucleare compotes, ad se suósque transtulerunt: Cæterùm hísce temerè consentire Analystas *Britannicos*, auctoritati alienæ ignavè succumbentes, pudet dolétque. Horum sanè adscribi ordinibus *Landen* patiamur forsàn, qui suis studiis, suísque opinionibus nimìs et pertinacitèr erat addictus, ita quòd *Analysin* suam *residualem*, (quâ tantoperè delectabatur,) ipsi *Methodo Fluxionum*, (cujus larva tantùm habenda sit, ut mox ostenditur,) arrogantèr et falsò anteposuerit; et qui, cum Analystæ, hujus ævi, celeberrimi, *D'Alembert* et *Euler*, adstipulantibus *Wildbore* et *Frisi*, communi consensu, ex diversis principiis, determinârant eandem veram quantitatem *motûs corporis circà axem liberè rotantis, ubi actione vis cujusdam extraneæ, aut percutientis aut accelerantis, perturbatur*—suam conclusionem, ab eorum conclusione non modicè dissentientem, eorum auctoritati quadruplici opponere ad mortem usque non desiit; pertinaciæ exemplum præbens, quòd maximis ingeniis haud rarò contingit, plus æquo "*sibi fidentibus aliâque spernentibus:*" At verò *censorem publicum*, apud M. R.; ipsúmque, ut ferunt, *Professorem Matheseos Academicum*, ita peccâsse, minimè condonandum est. Valdè equidem suspicor, illum, λογῳ αργῳ, "*ignavâ ratione*" seu negligentiâ, *ex nimiâ et præproperâ mentis festinatione*, hæc opinionum commenta

tentamina, valdè laboriosa in æquationibus altiorum demensionum, eruendæ, auspicatur; per inventionem *Limitum Æquationum* ex binomiis compluribus successivè substituendis, procedit; et per combinationem plurium *fractionum continuarum*, ex his operationibus elicitarum, concludit."

Quantâ autem comitate, quanto animi candore sustinuit *Vindicias* hásce *Newtonianas*, haùd temerè excitas, animadversionésque certè non leves; ex insequenti patebit epistolâ, quam mihi, homini obscuro, post acceptam *Analysin Æquationum*, scribere non dedignatus est vir clarissimus. Hanc verò, in scriniis meis hactenùs latentem, edere jam libet, minimè jactantiæ gratiâ,—studiis meis diù conversis, MATHESI relictâ, in MUSAS HÆBRÆAS—"*quarum sacra fero, ingenti perculsus amore*"—"smit with the love of *sacred song*"—sed quâ par est observantiâ; ne cuivis *Vindicias secundas* legenti nascatur suspicio, me, animo utcunque infenso, simultate quâvis, aut odio hostili, denegare velle grata "*præmia laudi*" tam insigni: quem ex peritissimis Newtoni successoribus, (excepto forsan *La Place*) maximísque *Algebræ puræ* et *Analyseos sublimioris* hoc ævo cultoribus, facilè principem, concedere cogor *Britannus*, et non inficiabitur æqua posteritas:

MONSIEUR, *Berlin*, *Nov.* 1. 1785.

Agréez mes très sincères remerciemens de l'honneur que vous m'avez fait en m'envoyant votre ANALYSE DES ÆQUATIONS; *que j'ai lue avec toute la satisfaction possible. La clarté et la précision qui regnent dans cet ouvrage, et la réunion qu'il présente des* METHODES ELEMENTAIRES *et des* THEORIES SUBLIMES *doivent lui donner un des premiers rangs parmi ceux de son genre; et par l'intérest que je prends aux progrès de la* GEOMETRIE, *je partage la reconnoissance que vos* COMPATRIOTES *vous doivent pour ce travail. Je desirerois fort pouvoir mériter l'opinion avantageuse que vous voulez bien avoir de moi, mais je ne puis y répondre, que par la véritable estimé, et la considération distinguée avec laquelle, J'ai l'honneur d'être,*

MONSIEUR,

Votre très humble et très obéissant serviteur,

DE LA GRANGE.

Neque existit ratio quævis, cur (in Mathematicis saltèm) animo infenso judiciis censorum eruditorum M. R. meum opponerem, qui satìs supérque laudabant ANALYSIN ÆQUATIONUM in recensione tractatûs illius, eodem circitèr anno; sed cum ambitiosè in lucem proferuntur *Exterorum* opera Mathematica—Cursus Mathematici *Sauri* et *Bezort*; Institutiones Analyticæ *Euleri* et *Clairaut, De La Place* et *De La Grange*; at *Nostratium*, fastidiosè in umbram relegantur, iniquâ forsan comparatione, à censoribus *Britannis*; cur non mihi æquo saltèm jure liceret vicissìm, afferre tanti censoris *Galli* testimonium, de *unâ* saltèm institutione elementariâ—etiàm *Hibernâ*?

ex aliis potiùs temerè assumpsisse: ídque vel ex meâ ipsius fortunâ in hâc re verisimile mihi videtur, cui contigit adolescenti *Rudimenta Fluxionum* ex *Maclaurin's Treatise of Fluxions* didicisse; et qui non nisi anno superiore (1798), tandem, cùm fortè curiosiùs ipsum textum *Newtonianum* inspexissem, atque ipsius auctoris simplicitatem elegantiámque demonstrandi, cum commentatoris prolixitate et ambagibus, contulissem, sententiam super hâc re mutare sum co-actus, et à commentatoris partibus in ipsius auctoris partes transire, et cum admiratione exclamare,

> *Homo homini quid præstet!*—Inventor
> Interpreti quanto antecellit!

24. Et mecum quidem et sentit et loquitur *Colson*, (Methodi Fluxionum *non sordidus auctor*) hísce verbis:

"The principles of the method here taught have been *scrupulously examined* and sifted, have been *vigorously opposed*, and, we may say, *ignorantly rejected* as insufficient, by *some* mathematical gentlemen, [*Berkeley*, &c.] who seem not to have derived their knowledge of them from *their only true source*; that is, our *author's own treatise*, written expressly to explain them."

25. Placet igitùr, resuscitatis hodiè objectionibus obsoletis, novísque ingruentibus, et per *Angliam*, orbémque literarum, à *Galliâ* grassantibus, totam methodum fluxionum ab ipsis fontibus, tàm *metaphysicis* quàm *mathematicis*, ab integro ad examen rigidum revocare; ut, his perspectis et per scrutinium investigatis, ne diutiùs falsis laboret criminibus evidentia ejus et certitudo; neque *injurioso pede proruant stantem columnam laudis* NEWTONIANÆ, vel imperiti vel invidi, sublimem et subtilissimam methodum ex sacris fontibus ANALYSEOS ANTIQUÆ haustam promotámque, quasi "*partum temporis*" abortivum, ex elementis incongruis conflatum, atque caducum, insimulantes, et non potiùs "*partum ingenii*," divinitùs conceptum, atque ad statum parturæ maturum atque perfectum, plenitudine temporis in lucem editum, *hæredem immortalitatis*, venerantes.

26. Nec minùs urget impellítque solicitudo, curáque non levis, NE QUID DETRIMENTI CAPIAT RESPUBLICA—influxu hoc infausto *opinionum Gallicarum*, tanto studio, tantísque illecebris, tantóque auctoritatis pondere, PHILOSOPHIAM BRITANNICAM invadentium; et nisi maturè et strenuè et peritè, "*zelo secundùm scientiam*" repellantur animosè, et rejiciantur indignantèr, IPSAM unà cum RELIGIONE ILLA, (cui modestè et sobriè hactenùs ancillari solita est) sub erroris et impietatis fluctibus obruere minitantium.

> ESTO PERPETUA!
> STATE SUPER VIAS ANTIQUAS!
> PRO FIDE SANCTIS COMMISSA, CONTENDITE!

27. Nec

27. Nec operam JUVENTUTI STUDIOSÆ, (noſtrati potiſſimùm) ingratam aut inutilem me præſtiturum ſpero, ſi ſtudia *Mathematica*, jamdudùm intermiſſa; recolens, SUBSIDIA QUÆDAM ELEMENTARIA ad *Phyſicam Newtonianam* (et præſertìm *Aſtronomiam*) PHILOSOPHIÆ PRIMÆ, SACRÍSQUE LITERIS, optimè conſentientem, pro virili, brevitèr, fidelitèr, et tamen ſimplicitèr perſpicuéque, proponere aggrediar:

"To illuſtrate *Newton* is to cultivate *real ſcience*; and to make his diſcoveries eaſy and familiar, will be no ſmall improvement in MATHEMATICS and PHILOSOPHY."——*Colſon.*

De Methodo Rationum Primarum et Ultimarum, ſeu Limitum.

28. TOTIUS Methodi Fluxionum fons atque origo eſt doctrina *Rationum Primarum et Ultimarum*, ſi Geometricè loquamur, ſeu *Limitum*, ſi Algebraïcè; ut rectè perhibent *D'Alembert* et *Cenſor* ille *profeſſorius*, Newtonum ipſum vel ſecuti, vel cum eo (inſcii forsàn,) conſentientes; qui ita loquitur:

——"Præmiſi verò *hæc Lemmata*, ut effugerem *tædium* deducendi longas demonſtrationes, more veterum Geometrarum, *ad abſurdum.* Contractiores enim redduntur demonſtrationes per *methodum indiviſibilium:* ſed quoniam durior eſt indiviſibilium hypotheſis, et propterea methodus illa *minùs geometrica* cenſetur, malui demonſtrationes rerum ſequentium ad *ultimas* quantitatum evaneſcentium *ſummas* et *rationes*, *primáſque* naſcentium; id eſt, ad *limites* ſummarum et rationum deducere; et propterea limitum illorum *demonſtrationes*, quâ potui brevitate, præmittere: his enim idem præſtatur quòd per methodum indiviſibilium; et *principiis demonſtratis jam tutiùs utemur.*"—"*Vímque* talium demonſtrationum ad methodum *præcedentium lemmatum* ſemper revocavi."

29. Pendet ſanè omnis hæc *Methodus Limitum*, ex *Lemmate primo*, *Lib.* I. *Princip.*

"QUANTITATES, *ut et* QUANTITATUM RATIONES *quæ ad æqualitatem tempore quovis finito conſtantèr tendunt*; *et ante finem temporis illius propiùs ad invicèm accedunt quàm pro datâ quâvis differentiâ*; *fiunt* ULTIMO ÆQUALES."

"Si negas, fiant *ultimò inæquales*; et ſit earum ultima differentia D: Ergò, nequeunt *propiùs ad æqualitatem* accedere, quàm pro datâ differentiâ D: Contrà hypotheſin." Q. E. D.

Ultimæ

Ultimæ rationes igitùr, ipso definiente *Newtono*, et quàm accuratissimè, et distinctissimè, sunt "*limites, ad quos quantitatum sine limite decrescentium rationes,* 1. *semper appropinquant*; et 2, *quas propiùs assequi possunt quàm pro datâ quâvis differentiâ*; 3. *nunquàm verò transgredi*; 4. *nequè priùs attingere quàm quantitates diminuuntur vel augentur in infinitum.*"

30. Sedulò autem et solicitè distinguit *Newtonus* inter harum quantitatum *magnitudines ultimas*, et *rationes ultimas*.—"In sequentibus igitùr si quando facili rerum conceptui consulens, dixero *quantitates quàm minimas*," vel "*evanescentes*" vel "*ultimas*," cave intelligas quantitates *magnitudine determinatas* sed cogita semper diminuendas *sine limite*."—Atque iterùm:—si quando quantitates tanquam *ex particulis constantes* consideravero, vel si pro *rectis* usurpavero *lineolas curvas*; nolim *indivisibilia* sed evanescentia *divisibilia*, non summas et rationes partium *determinatarum*, sed summarum et rationum *limites* intelligi.

31. Nec minùs solicitè diluit *Newtonus* objectiones impugnantium evidentiam et certitudinem doctrinæ limitum.

Objectio est "*Analystæ*" *(Berkley)*, quòd "*quantitatum evanescentium nulla sit ultima proportio*; quippe quæ *antequam* evanuerunt non est ultima; *ubi* evanuerunt, nulla est."—Sed et eodem argumento, regerit *Newtonus*, æquè contendi posset "*nullam esse corporis ad certum locum ubi motus finiatur pervenientis, velocitatem ultimam:* hanc enim, *antequam* corpus attingit locum, non esse ultimam; *ubi* attingit, nullum esse." At responsio facilis est: Per "*velocitatem ultimam*" intelligi, eam, quâ corpus movetur—neque antequam attingit locum ultimum et motus cessat, neque posteà—sed tunc, *cum* attingit; id est, illam ipsam velocitatem *quâcum* corpus attingit locum ultimum, et *quâcum* motus cessat. Et similitèr, per "*ultimam rationem quantitatum evanescentium*," intelligendam esse rationem quantitatum, non antequam evanescunt, non posteà, sed *quâcum* evanescunt. Paritèr et *ratio prima nascentium*, est ratio *quâcum* nascuntur. Et *summa prima et ultima*, est *quâcum esse* (vel *augeri* aut *minui*) incipiunt et cessant. Extat *limes* quem velocitas in fine motûs *attingere* potest, non autem *transgredi*; hæc est velocitas ultima: Et par est ratio *limitis quantitatum* et *proportionum omnium* incipientium et cessantium. Cùm hic limes sit certus et definitus, problema est *verè geometricum* eundem determinare. *Geometrica verò omnia in aliis Geometricis determinandis ac demonstrandis, legitimè usurpantur.*

32. "Contendi etiàm potest, quòd "*si dentur ultimæ quantitatum evanescentium rationes, dabuntur et ipsæ magnitudines ultimæ.*" Hæc verò consequentia non est necessaria. Nam fieri potest ut claram ideam habere possimus *relationis* ultimò intercedentis inter duas quantitates quam minimas, licet *absoluta* magnitudo ipsarum quantitatum nec inveniri nec concipi potest. Res clariùs intelligetur in infinitè magnis: si quantitates duæ quarum data est differentia, augeantur in infinitum, dabitur harum ratio ultima, nempè ratio æqualitatis; nec tamen idcircò dabuntur et ipsæ quantitates ultimæ, seu maximæ, quarum ista est

 ratio:

ratio: hæ enim, cùm in immenſum excreverint, omnem calculum, imò omnem *abſolutæ* magnitudinis conceptum, longè tranſiliunt et prorsùs refugiunt.

33. Illuſtrari poſſunt hæc, paulò ſubtiliùs forsàn et abſtractiùs diſputata quam pro tyronum captu, ex conſiderationibus *Algebräicis* et *Geometricis*, et *Arithmeticis*.

1. In reductione æquationum *quadraticarum*, *Analyſ. Æquat.* § 30. prodiit duplex valor radicis, nempè $x = \frac{1}{2}p \pm \sqrt{\frac{1}{4}p^2 - q}$, erítque valor uterque realis, ubi $\frac{1}{4}p^2$ fit *major* quam q; uterque verò impoſſibilis, ubi fit *minor*; et ipſo in *limite*, ubi $\frac{1}{4}p^2$ fit *æqualis* q, erit unicus valor radicis; evaneſcente jam quantitate radicali $\sqrt{\frac{1}{4}p^2 - q}$.

2. In reſolutione *cubicarum* quoque, § 307, ipſo in *limite*, ubi $\frac{3}{4}p^2$ fit *æqualis* q, duplex valor factoris quadratici, $x = -\frac{1}{2}p \pm \sqrt{-\frac{3}{4}p^2 + q}$ in unicum paritèr coaleſcit.

34. Et ipſe *Landen*, in demonſtrationem ſuam *Theorematis Binomii*, purè Algebräicam (ut refert) clanculùm introducit methodum *limitum*, ſeu fluxionum, ubi fingit quòd "*ſalvâ* æquatione *variari* poteſt y ad libitum, donec fiat $y = x$."—Analys. Æquat. p. 38.

Nec minùs *De La Grange*, naturam radicum æquationis optimè exponens, et theoriam æquationum Algebräicarum peritiſſimè promovens, ut docuimus Analys. Æquat. § 145—148, methodum *limitum* felicitèr ſanè introducit.

Hinc pendet omnis inventio *limitum* vel *radicum æquationis*, vel extremarum, vel intermediarum; quorum inventio inveſtigandis ipſis radicibus apprimè utilis eſt, per varias approximandi methodos.

35. In *ſectionibus conicis* quoque luculentiſſimè obtinet *methodus limitum*; inde enim pendet omnis theoria *circulorum* et *parabolarum* curvas alias *oſculantium*, ingenti cum commodo theoriæ motuum planetarum atque cometarum adhibita, à ſummis Analyſtis, *Newtono*, *Halley*, *Clairaut*, &c.

Nam, ſi concipiantur ſingulæ *ſectiones conicæ* gigni ex interſectionibus plani per axem coni perpetuò rotantis, cum ſuperficie conicâ vel ſuperficiebus conicis:

1. Sectio plani generantis, aut paralleli ad baſin coni, aut ſubcontrariè poſiti, erit *circulus*.

2. Circulus, rotante plano, ſtatìm migrabit in *ellipſin*; cujus eccentricitas, rotando, continuè augebitur, donec planum fiat parallelum lateri oppoſito coni ipſius; in hòc verò ſtatu mutationis, ellipſis viciſsìm, in *parabolam*, centro in infinitum abeunte, migrabit: deinde, rotante adhùc plano, parabola ſtatìm migrabit in *hyperbolam* atque *hyperbolas oppoſitas*, centro ſectionis jam extrinſecùs

tranſlato

tranflato inter vertices; et hyperbolarum eccentricitas augebitur, donec, plano cum ipfo axi tandem coincidente, hyperbolæ oppofitæ in *triangula verticalia* migrabunt. Atqui eadem contingent, fed ordine contrario, donec, totâ rotatione peractâ, reftituitur *circulus.*

3. Hinc conftat *circulum* effe limitem nafcentium ellipfium; *parabolam*, evanefcentium ellipfium et nafcentium hyperbolarum; et *triangulum*, evanefcentium hyperbolarum. Sed calculus limitum longè facilior prodit quàm calculus fectionum intermediarum; quoniam *circuli* omnes funt figuræ fimiles; necnon *parabolæ omnes*; fed fingulæ ellipfium vel hyperbolarum fpecies, rotando genitæ, funt inter fe diffimiles, et diverfos calculos, nempè, unaquæque fpecies calculum fibi proprium, requirunt.

36. Ex *rationibus arithmeticis*, hactenùs pofita quoque illuftrare fas eft.

1. *Si data quantitas* ($2a$) *dividatur inæqualitèr, exponente* (x) *partem intermediam variabilem; erit ratio facti fub partibus inæqualibus* $(a+x) \times (a-x)$ *ad quadratum femiffis, ratio minoris inæqualitatis*; propter defectum quadrati x^2. Per *Prop.* 5. *Lib.* 2. *Elem.*

Primò quidem, ratio *nafcentis* facti feu rectanguli, (quantitate quàm maximè inæqualitèr divisâ) ad hoc quadratum, ferè infinita evadit, propter x ferè æqualem a; dein, *decrefcente* x, ratio rectanguli ad quadratum perpetuò accedet ad æqualitatem; et ultimò, *evanefcente* x, ipfo in limite, fiet ratio æqualitatis.

Ex. gr. Sit $a = 101$, et primò, variabilis $x = 100$; erit rectangulum ad quadratum in ratione 201×1 ad 101×101; feu 201 ad 10201, feu 1 ad 50.7, hoc eft, in ratione longè minore. Dein, fit $x = 10$, et ratio rectanguli ad quadratum jam fiet 111×91 ad 101×101; feu 10101 ad 10201, feu 1 ad 1.009; propè ad æqualitatem accedente. Poftremò fit $x = 1$; evadétque ratio, 102×100 ad 101×101, feu 10200 ad 10201, feu 1 ad 1,000,09, proximè ad æqualitatem accedente.

37. "Res clariùs intelligetur in infinitè magnis."

Si eadem quantitas ($2a$) *perpetuò augeatur, defignante jam* x, *augmentum variabile, erit ratio rectanguli* $(2a+x) \times x$ *ad quadratum* $(a+x)^2$, *ratio quoque minoris inæqualitatis.* Propter defectum quadrati conftantis, a^2. Per *Prop.* 6. *Lib.* 2. *Elem.*

In hoc cafu quoque, *crefcente* x, ratio rectanguli ad quadratum, perpetuò ad æqualitatem accedit; et ultimò, *in infinitum aucto* x, fit ratio æqualitatis.

Ex. gr. Sit $a = 1$; et primò fit variabilis $x = 1$; erit rectangulum ad quadratum in ratione 3×1 ad 2×2; feu 3 ad 4, feu 1 ad 1.3333, &c. Dein, fit $x = 10$; evadétque ratio 12×10 ad 11×11; feu 120 ad 121, feu 1 ad 1.008; propiùs ad æqualitatem accedens: denuò fit $x = 100$; et fiet ratio 102×100 ad 101×101; feu 10200 ad 10201, feu 1 ad 1.000,08;

 longè

longè propiùs quàm in casu ultimo. Jam verò sit $x = 1000$; et ratio ejusdem rectanguli ad dictum quadratum quam proximè ad rationem æqualitatis accedet; erit enim æqualis rationi 1002×1000 ad 1001×1001, seu 1002000 ad 1002001, seu 1 ad 1,0000008. Denique sit valor x *infinitè magnus*; et fiet ultimò ratio rectanguli ad quadratum ratio æqualitatis; differentiâ constanti, a^2, $= 1$, tandem pro nihilo evadente respectu magnitudinum ipsarum adeò immensarum: hæc igitùr differentia *quasi* evanescit, licèt nunquam *actu* evanescit. Claram igitùr habere possimus ideam *relationis* inter duas quascunque magnitudines utcunque auctas vel diminutas in infinitum, licèt ipsarum magnitudinum *absolutarum* ideam determinatam seu positivam nullatenùs effingere fas sit, mortalibus, *finitas* tantùm *facultates* sortientibus.

38. Quò meliùs intelligatur natura *rationum primarum et ultimarum*, quam hìc pressiùs exposui; principia *Newtoniana*, brevissima et subtilissima quidem, quâ potui fidelitate evolvendo; placet insupèr sententias interpretum fusiùs adducere; non solùm illustrandi gratiâ, verùm etiàm ex abundanti confirmandi hactenùs exposita, atque obtrectatorum inscitiam plenissimè redarguendi, ex peritissimis *Newtoni* discipulis.

I.—*Colson's Account of the Method of Fluxions.*

39. "The *chief principle* upon which the Method of Fluxions is built is this very simple one, taken from the *Rational Mechanics*; which is, that *mathematical quantity*, particularly *extension*, may be conceived as generated by continued *local motion*; and that *all quantities* whatever, (at least by *analogy* and *accommodation*,) may be conceived as generated after a like manner: consequently, there must be *comparative* velocities [or rates] of increase and decrease during such generations; whose *relations* are fixed and determinable, and may therefore (problematically) be proposed to be found.

"This problem our author solves by the help of *another principle* not less evident; which supposes that quantity is *infinitely divisible*, or that it may (*mentally* at least) *so far* continually diminish, as at last, before it is *totally* extinguished, to arrive at quantities which may be called *vanishing* quantities, or which are *infinitely little*, and less than any *assignable* quantity: or it supposes that we may form a notion, not indeed of *absolute*, but of *relative* and *comparative infinity*.

"It is a very just exception to the Method of *Indivisibles* [of *Cavalerius*], as also to the foreign *Infinitesimal* Method [of *Leibnitz*], that they have recourse *at*

once to infinitely little quantities, and infinite orders and gradations of these, not relatively but absolutely such: they assume these quantities *simul et semel*, without any ceremony, as quantities that *actually* and *obviously* exist, and make computations with them accordingly; the result of which must needs be as *precarious* as the absolute existence of the quantities they assume.

" *Absolute* infinity, as such, can hardly be the object either of our conceptions or calculations; but *relative* infinity may, under a proper regulation. Our author observes this distinction very strictly, and introduces none but *infinitely little* quantities that are *relatively* so: which he arrives at, by beginning with finite quantities, and proceeding by a gradual and necessary progress of diminution: his computations always commence by *finite* and *intelligible* quantities; and then, at last, he inquires what will be the result, in *certain circumstances*, when such or such quantities are diminished *in infinitum*. This is a constant practice even in *Common Algebra* and *Geometry*; and is no more than descending from a *general proposition* to a *particular case* which is certainly included in it."

40. " And from these *easy* principles, managed with a vast deal of skill and sagacity, he deduces his *Method of Fluxions:* which if we consider only *so far* as he himself has carried it, together with the *application* he has made of it, either here or elsewhere, directly or indirectly, expressly or tacitly, to the most curious discoveries in *art* and *nature*, and to the sublimest *theories*, we may deservedly esteem it as the *greatest work of genius*, and as the noblest effort that ever was made by the human mind."

41. " Indeed it must be owned, that many useful *improvements* and *new applications* have been *since* made by others, and probably will be *still* made, every day: for it is no mean excellence of this method, that it is doubtless still *capable* of a greater degree of perfection, and will always afford an inexhaustible fund of curious matter, to reward the pains of the ingenious and industrious *analyst*."

42. " It should be well considered by these *gentlemen objectors*, that the great number of *examples* they will find here, to which the Method of Fluxions is successfully applied, are so many *vouchers* for the truth of the principles on which that method is founded: for the deductions are *always conformable* to what has been derived from other *uncontroverted* principles; and therefore must be acknowledged as *true*.

" This argument should have its due weight even with such as *cannot*, as well as with such as *will not* enter into the proof of the principles themselves. And the *hypothesis* that has been advanced to evade this conclusion—*of one error in reasoning being still corrected by another equal and contrary thereto*—and that, *so regularly*, *constantly*, *and frequently*, as it must be supposed to do here;—this hypothesis, I say, ought not to be seriously refuted, because I can hardly think it is seriously proposed."——*Preface to Colson's Method of Fluxions*, p. 11.

II.—*Mac-*

II.—*Maclaurin's Account of the Method of Fluxions.*

43. "We would not be underſtood to affirm that the Methods of *Indiviſibles* and *Infiniteſimals*, by which *ſo many uncontested truths* have been diſcovered, are without a foundation: we acknowledge further, that there is ſomething *marvellous* in the doctrine of *Infinites*, that is apt to pleaſe and tranſport us; and that the Method of *Infiniteſimals* has been proſecuted *of late*, with an acuteneſs and ſubtlety not to be paralleled in any other ſcience. But Geometry is beſt eſtabliſhed on *clear* and *plain principles*; and theſe ſpeculations are ever obnoxious to ſome difficulties: If the greateſt accuracy has been always required in this ſcience, in reaſoning concerning *finite* quantities, we apprehend that Geometricians cannot be *too ſcrupulous* in admitting or treating of *infinites*, of which our ideas are *ſo imperfect*.

44. "They who have made uſe of *infinites* and *infiniteſimals* with the greateſt liberty, have not agreed as to the *truth* and *reality* they would aſcribe to them.—*Cavalerius*, the ingenious author of the Method of *Indiviſibles*, diſcovered a method which he found to be of very extenſive uſe, and of an eaſy application for meaſuring and comparing planes and ſolids, and would not deprive the world of ſo valuable an invention. In propoſing it, he ſtrove to avoid the ſuppoſing magnitude to conſiſt of *indiviſible* parts, and to abſtract from the conſideration of *infinity*:—" Quoad continui compoſitionem, manifeſtum eſt ex præoſtenſis, ad ipſum *ex indiviſibilibus* componendum, nos minimè cogi: ſolùm enim *continuò ſequi indiviſibilium proportionem*, et *è converſo*; intentum fuit: quòd quidem cùm utráque poſitione ſtare poteſt. Tandem verò dicta *indiviſibilium aggregata* non ita pertractavimus, ut *infinitatis rationem* propter infinitas lineas ſeu plana ſubire videantur, &c."—*Cavalerii Geom. Indiviſ. Lib.* 7. *Præf.*——But he acknowledged that there remained *ſome difficulties* in this matter, which he was not able to ſolve:—and he ſpeaks as if he foreſaw that it ſhould be afterwards delivered in an unexceptionable form, which might ſatisfy the moſt ſcrupulous Geometrician; and leaves this *gordian knot* (as he expreſſes himſelf) to ſome *Alexander*."

45. "The celebrated Mr. *Leibnitz*, juſtly eſteemed for his various writings, owns *infinites* to be no more than *fictions*:—" On s'embaraſſe dans les ſéries des nombres qui vont à *l'infini*: On conçoit un *dernier terme*, un *nombre infini*, où *infiniment petit*: mais tout celà ne ſont que *des fictions*. Tout nombre eſt *fini* et *aſſignable*; toute ligne l'eſt le même."—*Eſſai de Théodicée*, diſc. prélim. § 70. —They who treated of infinites *before him* proceeded, as he obſerves, with a *timorouſneſs* which the contemplation of ſuch an object naturally inſpires:—

 "Quand

"Quand on y étoit arrivé, on s'arrestoit avec un espèce *d'effroy* et de *sainte horreur*—on regardoit *l'infini* comme un *mystère*, qu'il falloit *réspecter*, et qu'il *n'étoit pas permis d'approfondir.*"—"*They* stopt when they came to *infinity* with a sort of *holy dread*, and respected it as an *unfathomable mystery.*"—*He* adventures farther, in order to discover the source, and penetrate into the first principles of GEOMETRICAL truth. INFINITY, according to him, *is the great trunk from which its various branches are derived, and to which they all lead.* In this great pursuit, he displays *infinite* and *finite*, with a *freedom* that puts us in mind of the ancient poet [*Homer*] and his gods—whom he represents with *the passions of men*, and mingles in their *battles*."

"We doubt not that if a full and perfect account of all that is *most profound* in the HIGH GEOMETRY *could* have been deduced from the *Doctrine of Infinites*, it might have been expected from this author: but our *ideas of infinites* are too *obscure* and *inadequate* to answer this end: and there are many things advanced by *all those* who have applied them with great freedom in GEOMETRY, that give ground to a remark *like* to M. de *St. Evremond*'s, when he observes, that "it is surprising to find the *ancient poets* so scrupulous to preserve *probability* in actions purely *human*; and so ready to violate it, in representing the actions of *the Gods.*" —Some have not only admitted *infinites* and *infinitesimals* of *infinite* orders; but have distinguished even *nothings* into various kinds!—and if such liberties continue, it is not easy to foresee what *absurdities* may be advanced as *discoveries* in what is called the SUBLIME GEOMETRY."—"PHILOSOPHY probably will always have it's *mysteries*; but these are to be avoided in GEOMETRY, [as *far as is possible, and as much as in us lieth*]; and we ought to guard against abating from it's strictness and evidence the rather, that an ABSURD PHILOSOPHY *is the natural product of a* VITIATED GEOMETRY."

46. "Sir ISAAC NEWTON accomplished what CAVALERIUS had wished for, [and avoided the rock on which the daring *Leibnitz* had split] by inventing the *Method of Fluxions*; and proposing it in a way that admits of *strict demonstration*: which requires the supposition of no quantities but such as are *finite*, and easily conceived. The computations in this method are the same as in the Method of *Infinitesimals*; but it is founded on accurate principles agreeable to the ANCIENT GEOMETRY: In it, the *premisses* and *conclusions* are equally accurate; no quantities are rejected as *infinitely small* *, and no part of a *curve* is supposed to *coincide with a right line*. The excellency of this method has not been so *fully described*, or so *generally attended to*, as it seems to deserve; and it has been sometimes represented as on a level, in all these respects, with the Method of *Infinitesimals*. The chief design of this treatise, is to shew it's advantages in a clearer and fuller light, and to promote the design of the great inventor, by

* Compare *Newton*'s Demonstration of the *Fluxion of a Rectangle*, § 84, 85, with *Euler*'s, derived from the *Infinitesimal Method*, § 87, to which it is infinitely superior in accuracy and elegance.

establishing

establishing the HIGHER GEOMETRY on plain principles, perfectly consistent with each other, and with those of the *ancient Geometricians*."—*Maclaurin's Introduction to Fluxions.*

De Applicatione Methodi Rationum Primarum et Ultimarum seu Limitum, ad Quadraturam Curvarum, atque ad Mechanicam Rationalem.

47. HUJUS applicationis elegantia quædam proferuntur exempla à *Newtono* in *Lemmatibus insequentibus:* LEM. II. et III. *Si polygonorum rectilineorum figuræ cuivis curvilineæ similiter inscriptorum et circumscriptorum augeatur numerus laterum correspondentium, et diminuatur magnitudo in infinitum, ità ut differentia polygoni inscripti et circumscripti tandem fiat minor quâvis datâ: ultimæ rationes quas habent polygonum inscriptum et circumscriptum, ad se invicem, (et multò magis ad figuram curvilineam intermediam) erunt rationes æqualitatis.*

Immediatè et nullo negotio sequitur hoc, ex *Lemmate primo* fundamentali; ídque sive polygonorum latera sint *æqualia*, ut in *Lemmate secundo*; sive *inæqualia*, ut in *Lemmate tertio.*

Cor. 1. Hínc *polygona inscripta et circumscripta*, tandem evanescente differentiâ, *ultimò coincident omni ex parte, cum figurâ curvilineâ.*

Cor. 2. Et propterea, *polygona ultima*, quoad perimetros suos, non sunt *rectilinea*, sed rectilineorum *limites curvilinei.*

48. Atqui hæc est ipsa *Methodus Exhaustionum* quam exercebant *Geometræ veteres*, in compendium redacta, et longè promota, ab hoc artifice summo; sicut luculentiùs ostenditur in *Introductione Maclaurinianâ:*

"The *Method of Exhaustions* was that by which the Ancients demonstrated all their theorems for measuring and comparing *curvilinear* figures. It is often said that *curve lines* have been considered by them as *polygons of an infinite number of sides*; but this principle no-where appears in their writings: we never find *them* resolving any figure or solid into *infinitely small elements:* on the contrary, they seemed to avoid such suppositions; as if they judged them *unfit* to be received into GEOMETRY, when it was obvious that their demonstrations might have been sometimes *abridged* by admitting them."

"They considered *curvilinear areas* as the *limits* of circumscribed or inscribed *figures* of a *more simple kind*, which *approach* to these limits (by a bisection of lines

lines or angles that is continued at pleasure); *so that the difference betwixt them may become less than any given quantity:* The inscribed or circumscribed figures were always conceived to be of a number and magnitude that is *assignable*; and from what had been shewn of *these figures*, they demonstrated the mensuration or the proportions of the *curvilinear limits themselves*, by arguments *ab absurdo*. They had made frequent use of demonstrations of this kind from the beginning of the ELEMENTS [as where the affections of *similar polygons*, found in the first six books, are transferred to *circles* in the twelfth book.] And these are in a particular manner adapted for making a transition from *right-lined* figures to such as are bounded by *curve lines*. By admitting *them* only, they established the more difficult and sublime parts of their *Geometry*, on the same foundation as the first elements of the science. *Nor could they have proposed to themselves a more perfect model.*"

49. "We have already observed how solicitous *Archimedes* appears to be, that his demonstrations should be found to depend on those principles only that had been *universally received* before his time. In his Treatise of the *Quadrature of the Parabola*, he treats of a progression whose terms decrease constantly in the proportion of 4 to 1 (which we expressed by the trapezia CD*ba*, DG*cb*, GR*dc*); but he does not suppose this progression to be continued to infinity, or mention the sum of an infinite number of terms; though it is manifest, that all which *can* be understood by those who assign that sum, was *fully* known to him. He contented himself with demonstrating this plain property of such a progression: That *the sums of the terms continued at pleasure, added to the third part of the last term, amount always to four-thirds of the first term:* (as, for instance, in the *Quadrature* of the *Parabola*, *Introduct.* p. 27—29. Pl. 3. *Fig.* 13. the sum of the trapezia CD*ba*, DG*cb*, GR*dc*, added to one-third part of the last trapezium GR*dc*, amounts always to four-thirds of the first trapezium CR*da*, or to it's limit, the triangle CA*a*;) but while this first trapezium approaches continually to it's limit, the inscribed polygon at the same time is approaching to the *parabolic* segment it's limit; and these are equal to each other, as there demonstrated, or in *Hamilton*'s *Conic Sections*, (*De Quadraturâ Parabolæ*, according to *Archimedes*.)—After all the methods that have been proposed for demonstrating the Quadrature of the Parabola, this *of the inventor himself seems to have a particular accuracy and elegance:* he does not suppose the chords of the curve to be bisected to infinity; so that, after an infinite bisection, the *inscribed polygon* might be said to *coincide* with the *parabola*. These suppositions had been new to the geometricians in his time; and such he appears to have avoided carefully.—On this occasion, he shews *how to find* the sum of any *number of terms that decrease constantly in the proportion of four to one:* and by this example of a *geometrical progression*, (as it is commonly called,) opened up a subject, which has been treated at great length by the modern geometricians."

50. "But

50. " But even *Archimedes* himſelf has not eſcaped the cenſures of ſome writers, *Hobbes*, *Joſeph Scaliger*, &c. who, being *unſkilful* in Geometry, and *unable to reconcile their own conceits with his demonſtrations*, have repreſented him as in an error, and " miſleading mathematicians by his *authority*."—*Plutarch*, however, celebrates the *ſimplicity* and *plainneſs* with which he treats the moſt difficult and abſtruſe queſtions :

Ου γαρ εςιν ε; Γεομετρια χαλεπωτερας και βαρυτερας ὑποθεσεις εν ἁπλεςεροις λαβειν και καθαρωτεροις ςοιχειοις γραφομενας.

" *For it is impoſſible to find more difficult and weighty ſubjects written in ſimpler and purer elements.*"

" He appears indeed to have been more fond of preſerving to the ſcience all it's *accuracy* and *evidence*, than of advancing *paradoxes*."

" And although the purſuit of *general* and *eaſy* methods may have induced ſome moderns to make uſe of *exceptionable* principles, and the vaſt extent which the ſcience has of late acquired, may have occaſioned their propoſing incompleat demonſtrations, yet the *firſt eſſays* were deduced from a careful attention to his ſteps :—" *C'étoit en obſervant de près la marche d'* ARCHIMEDE, *qu'il* (M. De Roberval) *étoit arrivé à cette ſublime et merveilleuſe ſcience*."—*Acad. Scient.* an. 1693.—This is generally acknowledged by the writers of that time."

51. " Hence it was with caution that the Doctrine of *Indiviſibles* was at firſt employed in Geometry by *Cavalerius*; therefore he ſubjoined more *unexceptionable* demonſtrations to thoſe he had deduced from his own principles : and the diſputes which enſued (the firſt of any moment that were known between *geometricians*) juſtified his precautions."

52. " Geometricians of the firſt rank had recourſe to this method of *limits* long ago, on ſeveral occaſions, as a method of the ſtricteſt kind : M. de *Fermat*, in a letter to *Gaſſendi*, and M. *Huygens*, in his *Horologium Oſcillatorium*, have employed it for compleating the demonſtrations of ſome theorems that were propoſed by *Galileo*, and proved by him in a leſs accurate manner; and Dr. *Barrow* has demonſtrated by it a theorem concerning the *Tangents of Curve Lines*."

53. In *Quarto Lemmate* demonſtrat Newtonus *rationes limitum curvilineorum, ultimò eſſe eaſdem ac polygonorum inſcriptorum.*

Nam ut *partes ſingulæ* polygonorum inſcriptorum *ad ſingulas*, ita (componendo) fit *ſumma omnium* ad *ſummam omnium*; et proindè (per *Lem.* 3. *Cor.* 2. præced.) *limes curvilineus ad limitem curvilineum.* Q. E. D.

Cor. Hinc, ſi *duæ cujuſcunque generis quantitates*, in *eundem partium numerum* utcunque dividantur; et partes illæ, ubi numerus earum augetur et magnitudo diminuitur

diminuitur in infinitum *datam obtineant ad invicem rationem*, prima ad primam, secunda ad secundam, cæteræque suo ordine ad cæteras: erunt *tota* ad invicem *in eâdem illâ datâ ratione*. Nam, si sumantur tota inter se ut partes, erit totum prius ad summam partium, et totum posterius ad summam partium, in ratione æqualitatis; et proindè, per hypothesin, in ultima ratione partis ad partem.

Q. E. D.

54. Ope hujus *principii generalissimi*, tàm GEOMETRIAM quàm ALGEBRAM permeantis, emendare licet demonstrationem *Tacquetianam*, *Prop*. 1. *Lib*. 6. *El*. Nempè "*triangula æquialta*, &c. *esse ad invicem in ultimâ ratione basium*;" quæ deficit, ubi de *incommensurabilibus* agitur; cum "*indicium proportionis*" pro *Euclidiano* malè substitutum à *Tacquet*, *commensurabilibus* tantùm restringitur.

Sumptis partibus similibus DEG et DG trianguli prioris DEF, et basis prioris DF; si substituantur hæ partes in triangulum posterius ABC et basin posteriorem AC respectivè, quoties fieri potest; ubi numerus partium augetur et magnitudo diminuitur in infinitum, ultimò *datam obtinebunt ad invicem rationem*: et proindè, erunt *tota* ad invicem in *eâdem illâ datâ ratione*: hoc est, erit triangulum prius DEF ad posterius ABC, ut basis prior DF ad posteriorem AC. Q. E. D.

55. Quàm iniquæ igitùr sunt in *Newtonum* criminationes, quasi "*methodum recentem*," "veteribus *ignotam*," "*alienis* principiis fundatam, à doctrinâ *motûs* translatis," iisdémque "*minimè necessariis*" excogitasset!—Pudet equidèm tales cavillationes unquam admisisse analystas ex peritissimis *Landen*, *D'Alembert*, et *Censorem* illum *Professorium*, præ incuriâ et oscitantiâ saltèm, si non ex invidiâ oriundas. Maximè autem pudet viri clarissimi *De La Grange*:—nonne *dormitat bonus*, ubi profitetur "se traditurum *functionum analyticarum* theoriam, ab omni consideratione *limitum* immunem!"—(si fides habenda sit isti *Censori* apud THE MONTHLY REVIEW.)

Dic quibus in terris——
——*et eris mihi magnus Apollo!*

56. Atqui luce clariùs constat, ex his documentis, principia infirma et vacillantia *methodi differentialis*, quæ primò posuerat *Leibnitz*, ex notionibus suis vagis et incongruis de *infinitesimis*, (seu quantitatibus infinitè parvis) gradatìm et lentè purgâsse, et tacitè, discipulos suos; atque ad *Newtonianam veritatem*, atque *rigorem Geometricum*, tandem et quasi proprio jure, perfecisse; nunc verò reclamare audent ingrati, debito suo honore *Newtonum*, successorem dignum veterum analystarum, *Archimedis*, *Apollonii* et *Euclidis*, iniquissimè defraudantes!

57. Demonstrârat *Euclides*, *Prop*. 20. *Lib*. 6. *El*. quod "*Polygonorum similium latera omnia quæ sibi mutuò respondent sunt proportionalia*, et, 2. *Areæ sunt in duplicatâ ratione laterum*:" hanc proprietatem insignem ad *circulum* transtulit *Archimedes*;

Archimedes; et ipse Euclides in *Prop.* 2. *Lib.* 12. *El.* Et generalitèr, extendit *Newtonus* ad *curvas quascunque*, in *Lemmate quinto*; eodem ratiocinio posito, ídque, sive *latera* sint *curvilinea*, sive *rectilinea*.

58. In *Lemmate septimo*, demonstrat *Newtonus*, (per evanescentiam anguli contactus, *Lem.* 6.) quod *ultima ratio arcûs quam minimi*, *chordæ* et *tangentis est ratio æqualitatis*; paritérque *semiarcûs*, *ordinatæ* et *abscissæ tangentis*; vel *chordæ semiarcûs*, *ordinatæ*, &c. Et propterea *hæ omnes lineæ* in omni de *rationibus ultimis* argumentatione pro se invicèm usurpari possunt.

59. In *Lemmate octavo*, quod *ultima forma triangulorum evanescentium*, (ubi ex rectis à centro quodam quasi vertice, ad basin productis, sive hæc basis, sit vel arcus, vel chorda vel tangens, fiunt hæc triangula) *est similitudinis*, et *ultima ratio æqualitatis*. Et hinc *triangula illa* in omni de *rationibus ultimis* argumentatione pro se invicèm usurpari possunt.

60. In *Lemmate nono*, demonstrat (per *Lem. quintum*), *quod areæ triangulorum curvilineorum similium* ex curvarum tangentibus seu secantibus, et *ordinatis* seu subtensis externis angulorum contactûs factorum, *erunt ultimò ad invicèm in duplicatâ ratione laterum*.

61. Elegantissimâ translatione factâ ad *Mechanicam Rationalem*, in *Lemmate decimo* demonstrat, quòd *si corpus, urgente vi quâcunque finitâ, moveatur*; sive, 1. *vis illa determinata et immutabilis sit*; sive, 2. *eadem continuò augeatur vel continuò diminuatur*; *erunt spatia ipso motûs initio in duplicatâ ratione temporum*.

Nam, si *tempora* motûs exponantur per *abscissas*, et *velocitates genitæ* per *ordinatas externas*, (seu subtensas angulorum contactûs) spatia, hâc vi urgente, descripta ritè exponentur per *areas triangulares* curvilineas hísce ordinatis descriptas; et proinde (per *Lem.* 5 et 9.), erunt in duplicatâ ratione laterum correspondentium, h. e. *abscissarum* seu *temporum*.

Cor. 1. Et quoniam virium diversarum intensitates ritè exponuntur per *velocitates* quas generant, erunt *hæ vires*, ut *spatia*, ipso motûs initio descripta directè et *quadrata temporum* inversè; per *Prop.* 23. *Lib.* 6. *El.*

62. In *Lemmate undecimo* demonstrat *Newtonus*, quòd *ordinata externa* (seu *subtensa evanescens anguli contactûs) in curvis omnibus curvaturam finitam ad punctum contactûs habentibus, est ultimò in ratione duplicatâ chordæ arcûs contermini*.

Nam, ex naturâ *circulorum æquicurvorum*, circà has areas triangulares descriptorum, erunt *ordinatæ externæ*, seu *circulorum sagittæ*, his proportionales, ut quadrata *chordarum* arcuum conterminorum directè, et *diametri* circulorum inversè; sed, ultima ratio diametrorum, evanescente differentiâ, fit ratio æqualitatis; adeóque ordinatæ externæ erunt ut quadrata chordarum.

E   *Cor.* 1.

Cor. 1. Hinc, *sagittæ curvarum* quoque, quæ *chordas arcuum duplorum* bisecant, et ad datum centrum motûs convergunt, *sunt etiàm ultimò ut quadrata arcuum conterminorum.* Nam *sagittæ illæ* sunt ut *ordinatæ externæ*, et *arcus* cum *chordis* ultimò coincident, per *Lem.* 7.

Cor. 2. Ideóque *sagittæ illæ sunt in duplicatâ ratione temporum quibus corpus datâ velocitate describit arcus quam minimos.* Nam datâ velocitate, hi arcus sunt ut tempora; et proindè quadrata arcuum ut quadrata temporum.

63. Atque sunt hæc *principia mathematica generalissima*, quorum ope demonstrat *Newtonus* in sequentibus propositionibus, *Sect.* 11. *rationes virium centripetarum* ad invicèm; *legésque* secundùm quas corpora circà commune centrum motûs revolventia, accelerantur aut retardantur pro diversis à centro distantiis. Vide § 88, 89 et 103.

De Fluxionum Elementis Primis.

---

64. Quo magìs pateat hujúsce Methodi, sive *Fluxionalis* sive *Differentialis*, evidentia et certitudo, visum est "*fundamentum* totius methodi" inspicere et perscrutari, quale illud struxit artifex ille egregius in *Lemmate secundo Lib.* 2. *Princip.* ulteriúsque obitèr exposuit, in *Analysi per Series Numero Terminorum Infinitas*, atque in *Quadraturâ Curvarum*;—quæ tria opera "*chordam triplicem haud facilè disrumpendam*" constituunt, vel viribus obtrectatorum *Britannorum* vel *Gallorum*, vel aliorum quorumvis contrà hanc doctrinam infaustè combinatorum.

65. Terminos fluxionum *technicos* scitè definit *Newtonus*, et expositione satìs dilucidâ distinguit:

1. "*Genitam* voco, quantitatem omnem quæ ex lateribus vel terminis quibuscunque, in Arithmetica per multiplicationem, divisionem et extractionem radicum; in Geometria, per inventionem vel contentorum et laterum, vel extremarum et mediarum proportionalium, sine additione et subductione generantur: ejusmodi quantitates sunt *facti*, *quoti*, *radices*; *rectangula*, *quadrata*, *cubi*, *latera quadrata*, *latera cubica*, et similes. Has quantitates, *ut indeterminatas* et instabiles, et *quasi motu fluxúve perpetuo* crescentes vel decrescentes, hìc considero: et

2. "Earum incrementa vel decrementa momentanea, sub nomine *momentorum* intelligo; ita ut *Incrementa*, pro *momentis addititiis* seu *affirmativis*; ac *Decrementa*, pro *subductitiis* seu *negativis*, habeantur.

 66. "Cave

66. "Cave tamen intellexeris [ſub nomine *momentorum*] *particulas finitas*: particulæ finitæ non ſunt *momenta*, ſed *quantitates ipſæ* ex momentis genitæ; [per vocem *momenta*] intelligenda ſunt *principia jamjam naſcentia* finitarum magnitudinum. Neque enim ſpectatur in hoc lemmate *magnitudo* momentorum, ſed *prima naſcentium proportio*."

Idem quoque fuſiùs exponit in *Analyſi per Æquationes*; doctiſſimo *Colſon* ita Anglicè eam reddente:

"The *moments* of *flowing* quantities are their *indefinitely ſmall* parts, by the acceſſions of which, in *indefinitely ſmall portions of time*, they are continually increaſed."

3. "Now theſe quantities, which I conſider as gradually and indefinitely increaſing, I ſhall hereafter call *Fluents*, or flowing quantities."

67. 4. "Whereas only quantities of *the ſame kind* can be compared together, and alſo their *velocities* [or *rates*] of increaſe or decreaſe; therefore, in what follows, I ſhall have no regard to *time*, formally conſidered; but I ſhall ſuppoſe *ſome one* of the quantities propoſed (being of the ſame kind) to be increaſed by an *equable* quantity, to which *the reſt* may be referred, *as it were*, to *time*: and therefore, by way of *analogy*, it may not improperly receive the name of *Time*. When, therefore, the word *time* occurs in what follows, (which, for the ſake of *perſpicuity* and *diſtinction*, I have ſometimes uſed,) by that word, I would not have it underſtood as if I meant time in its *formal* acceptation, but only that *other quantity*, by the equable increaſe or flux whereof time is *expounded* and *meaſured*."

68. 5. "Eodem recidit, ſi loco *momentorum* [ipſorum,] uſurpentur vel *velocitates* incrementorum ac decrementorum, (quas etiàm, "*motûs mutationes*," et "*fluxiones* quantitatum," nominare licet,) vel *finitæ quævis quantitates*, velocitatibus hiſce *proportionales*."

Et paritèr quoque, ſcribit *Newtonus* in *Quadraturâ Curvarum*:—"*Fluxiones* ſunt quàm proximè ut fluentium *augmenta æqualibus temporibus quàm minimis genita*; et, ut accuratè loquar, ſunt *in primâ ratione momentorum naſcentium*."

And again, in the *Analyſis*:—"The *moments* of flowing quantities—are *as* the *velocities* [or *rates*] of their flowing or increaſing;"—"and the *velocities* [or *rates*] by which every fluent is increaſed, by it's generating motion [whatever that may be], I may call *Fluxions*; or ſimply, *Velocities* or *Celerities*."

69. Ex his documentis undique congeſtis et intertextis, luculentiſſimè conſtat, quantâ peritiâ, ſolicitudine et cautione, methodi ſuæ generalis fundamentum poſuit Newtonus, in *definitionibus*, *diſtinctionibúſque* accuratiſſimis; "*perſpicuitatis*" (ut ipſe dicit) quàm maximè ſtudioſus, in re longè difficillimâ et ſubtiliſſimâ.

liſſimâ. Abundè autem liquet, eum uſurpâſſe vocem "*velocities*," (quæ tantas tragœdias excitavit), in latiore ſenſu, pro *rationibus* creſcendi vel decreſcendi—"*rates* of increaſe or decreaſe."—Et inde ſenſu generaliſſimo definiri poſſunt *fluxiones*—*Menſuræ quævis rationum ſecundùm quas quantitates fluentes quælibet ejuſdem generis, quolibet in ſtatu mutationis, utcunque variantur.*

70. Quidni autem eadem licentia *Newtono* concedatur, quam ſibi ſumebant *Analyſtæ veteres?* Cur non, æquo ſaltèm jure, *Newtono* fas fuerit, in rebus geometricis principium geometricum adhibere, et "*genitas*" quaſcunque quantitates, ex fluxu, ſeu "*quaſi motu perpetuo*," oriundas, conſiderare; quàm *Archimedi*, *Apollonio*, et *Euclidi*, concipiendi *lineam* quaſi ex motu *puncti*, *ſuperficiem*, quaſi ex motu parallelo *lineæ*, et *ſolidum*, quaſi ex motu parallelo *ſuperficiei*, reſpectivè genita? ídque, *illuſtrandi* gratiâ, nimirùm, in rebus *metaphyſicis* à ſenſibilibus valdè abſtractis? Nonne Geometria ipſa, exiguos fines *definitionis nominis* aſpernans, longè, imò longiſſimè, ultrà "*menſurationem terræ, ſive agrorum*,"—"*land-ſurveying*"—in remotiſſimas cœlorum regiones, ultrà orbem *planetarum*, *cometarum* atque *aſtrorum*, ad ipſam *viam lacteam* excurrit—fræni impatiens, "nec audit habenas?" Et quidnì vocabulum "*fluxio*," quoque, in ſenſu latiſſimo atque ampliſſimo, ad omnigenas *functiones*, MECHANICÆ RATIONALIS,—*vires*, *pondera*, *denſitates*, *reſiſtentias*, *reflexiones*, *refractiones*, *infractiones*, *aberrationes*, &c. ſeu quaſcunque quantitatum mutationes, ſecundùm magnitudines et directiones, ritè exponendas, extendatur et amplietur?—De *definitionibus nominum* diſputare, (non κατα φυσιν ſeu "*ſecundùm rei ipſius naturam*," ſed κατα θεσιν, "*pro definientis arbitrio*"—vel *inſtituto*)—puſilli eſt ingenii, *minutíque philoſophi.*—Eodem redeunt *Fluxiones* Newtoni et *Functiones* De La Grange; ut ex definitione vocis poſterioris, apud The Monthly Review, conſtabit:

71. "A *function* of one or more quantities, is *every expreſſion* in which theſe quantities enter *after any manner whatever* (combined, or not, with other quantities of given or variable values), *while* the quantities of the *function* may receive *all poſſible values.*"

72. Eodem quoque recidit *Methodus Differentialis*, (definiente analyſtâ peritiſſimo, *Euler*, in ſuis *Inſtitutionibus*) ac ipsa *Fluxionalis*:

"*Methodus determinandi rationem incrementorum evaneſcentium, quæ functiones quæcunque accipiunt*; DUM *quantitati variabili cujus ſunt functiones, incrementum evaneſcens tribuitur.*"

Velùt ſi quantitatis variabilis x, incrementum fit i; ubì hæc quantitas fit $x+i$, fiet hujus quadratum, $x^2+2xi+i^2$. Erit igitùr

$$\textit{Increm. } x : \textit{increm. } x^2 :: i : 2xi + i^2;$$

$$\text{vel, dividendo per } i, \text{ ——— } :: 1 : 2x + i.$$

Et

Et proindè ratio 1 ad $2x$, exponet rationem incrementi lateris variabilis x ad incrementum quadrati variabilis x^2, ipso in *limite*, ubi *i* evanefcit, accuratè; antequàm evanefcit, quàm proximè tantùm. At *limes rationis horum incrementorum evanefcentium* folummodò in *Calculo Differentiali*, *Eulero* judice, fpectatur.

73. Quis non videt analyfta benè cordatus, ad amuffim quadrare Methodum *Fluxionalem* cum *Methodo Differentiali*, tàm *Euleri* quàm *De La Grange?*—et methodo utrique æquè competere confiderationem *temporis* inter crefcendum decrefcendúmve elapfi?—fi non "*formalitèr*" quidem, faltèm per *analogiam*, five per *accommodationem*, et illuftrandi gratiâ?—et quale aut quantum intercedat difcrimen (intelligo verum difcrimen, non nomini tenùs) inter "*momenta*" Newtoni, et "*incrementa evanefcentia*" Euleri, detegat, oro, analyftarum vel perfpicaciffimorum curiofa felicitas, in fæcula fæculorum.

74. Non idcircò nifi imperitè objicientis eft "*animadverfio* docti *Torelli*," quam commendat M. R. quafi—*inter exempla admodùm perfpicuæ validaéque argumentationis*" annumerandam:

"*Neque verò quidquam agunt, qui*, infinitefimas quantitates *rejicientes*, momenta quædam *earum loco* GEOMETRIÆ *inferunt*; *eáque ita vocant, quòd concipiantur* momento temporis *finitæ magnitudinis fluxu oriri: Si enim* momenta hæc *vires fuas fluendo explicant, non aliud funt quàm* ipfæ *quas rejiciunt* infinitefimæ quantitates; *fin autèm nituntur neque tamen ultrà tendunt*, novum vocabulum *fruftrà inducitur*, *dum id quod* nihil *eft* momentum *appellatur*."

At ruit objectio fuis viribus everfa. Nam, 1. *momentum* non eft *nihil*, ut ex definitione conftat; nec, 2. eft *infinitefima quantitas*, cum *magnitudinis finitæ*, feu determinatæ, (licet *quàm minimæ*) effe concipiatur; nam, ut accuratiffimè loquitur *Newtonus*, eft "*augmentum fluentis tempore quàm minimo genitum*;" neque, 3. fi valida fit, minùs hoftilitèr in Methodum *Differentialem*, quàm *Fluxionalem*, invehitur, iifdem principiis innixas; totius analyfeos fublimioris fundamentum æquè labefactando.

75. Nec minùs ruit altera ifta objectio *D'Alembert* (*f*), in vocem "*Velocitatem*" animadvertentis, quippè quæ eodem redit in fenfu *Algebräico*, ac "*ratio*:"

(*f*) "To fpeak of *D'Alembert* merely as a *mathematician*," THE MONTHLY REVIEW feems rather to over-rate his merits:—However profound his refearches, and extenfive his labours "to clear away the rubbifh of *jargon* and *myftery* from fcience," his calculations and demonftrations are exceedingly rugged, prolix and intricate; far removed from the elegance, concifenefs and arrangement of *Newton*, and even of *Euler* and *Clairaut*.—When *Landen* (himfelf a profound mathematician, and who firft difcovered the fource of *Newton*'s miftake, in determining the quantity of the *preceffion of the equinoxes*, the merit of which is attributed to *D'Alembert* by the M. R.) had examined *D'Alembert*'s folution of the *Theory of Rotatory Motion*, differing from his own, he was unable to difcover the fource of that difference in the *obfcurity* of his argument, until affifted by the perfpicuity of *Euler*'s manner; whofe folution agreed in it's refult with *D'Alembert*'s.—And furely his objection to the Method of Fluxions fhews that he was not deeply verfed in the *metaphyfique* of the fcience.

et in ſenſu *Geometrico*, legitimè pro *fluxione* uſurpanda eſt, cum *velocitas* ſit "*exponens rationis*" inter ſpatium et tempus intercedentis, ubi motus ſit uniformis; et proinde *relationem* verè exiſtentem inter veras quantitates commenſurabiles in *Mechanicâ Rationali*, aptè, et "*naturalitèr* quidem," (ipſo fatente M. R. p. 498.) deſignat. Et ſecundùm *D'Alembert*—*Calculi Differentialis* in Geometricis, munus eſt—"*déterminer algebräiquément* LA LIMITE *d'un rapport de laquelle on à déjà l'expreſſion en lignes*; *et à égaler ces* deux limites; *ce qui fait trouver* une des lignes *qu'on cherche*."

Atque illuſtrandi gratiâ, ductâ tangente *parabolæ*, cujus æquatio eſt $ax = y^2$, primò exquirit *D'Alembert* valorem unius limitis algebräicè, $(\frac{a}{2y})$; et valorem alterius geometricè $(\frac{y}{s})$; quibus æquatis, deducit valorem *ſubtangentis*, $(s) = \frac{2y^2}{a} = 2x$. Atque agnoſcit paritèr, (ut, ex abundanti diluatur objectio *Torelliana*) quòd in formulâ $\frac{dx}{dz}$, ſecundùm notationem differentialem, ſeu $\frac{\dot{x}}{\dot{z}}$ ſecundùm fluxionalem), dx et dz (ſeu $\dot{x}$ et $\dot{z}$) non deſignare *quantitates infiniteſimas*, ſeu infinitè parvas, ſed in certo quodam ſtatu *finitæ magnitudinis*; tales enim *ſymbolicè* tantùm uſurpari pro formâ loquendi compendioſâ, calculis concinnandis accommodatâ, verùm nullatenùs *criticè*, ſeu pro *uſu philoſophico* terminorum technicorum.

76. Atque hoc pacto, ipſe quoque *Cenſor*, p. 487, monet, quòd ſi *curvæ cujuſdam axis* denotet *tempus* deſcriptionis; et *ordinatìm applicatæ* quibuſvis curvæ ipſius punctis, *ſpatia* (vel actu deſcripta vel quæ deſcribenda forent velocitate exinde uniformitèr continuatâ); erit *velocitas* ubivis, ut *quotiens ordinatæ per ſubtangentem diviſæ*, ſeu $v = \frac{o}{s}$: et proindè per hanc rationem *verè* exponitur ſeu menſuratur.

77. Hactenùs explicatis elementis primis Methodi Fluxionum, pro ſuâ ſubtilitate fuſiùs; facilitati conceptûs conſulens, et ſpeculationibus *metaphyſicis* facem præbens, quantum res difficillima pateretur; et abſolutis *vindiciis* adverſùs obtrectatores laudum *Newtonianarum*, tàm exteros quàm noſtrates; gratiam ut ſpero, promeriturus *omnium* cordatiorum analyſtarum, qui quantum *Newtono* debent agnoſcere haud gravantur; (*Anglorum* potiſſimùm, ingens illud decus BRITANNIÆ, atque Analyſeos ſublimioris principem et architectum, optimo jure admirantium): Supereſt ut ipſius methodi generalis *rudimenta* proponam; preſſiùs atque concinniùs, et tamen uberiùs ac luculentiùs, quàm alii

alii ſcriptores qui hactenùs hoc facere aggreſſi ſunt, hâc, nimirùm, lege mihi propoſitâ et ſedulò obſervatâ, ut veſtigiis ipſius inventoris ſagaciſſimi, quàm maximè potuerim, fidelitèr ſemper inſiſterem ;

Te ſequor, O MAGNÆ GENTIS DECUS ! *inque tuis nunc*
Fixa pedum pono preſſis veſtigia ſignis ;
Non ita certandi cupidus, quàm propter amorem,
Quòd te imitari aveo :——
E tenebris tantis tam clarum extollere lumen
Qui primus potuiſti, illuſtrans commoda vitæ.—
Tu PATER ! ET RERUM INVENTOR !——
————*Tuiſque ex,* INCLUTE, *chartis,*
Floriferis ut APES *in ſaltibus omnia libant,*
Omnia NOS *itidem depaſcimur aurea dicta,*
Aurea, perpetuâ ſemper digniſſima vitâ. LUCRETIUS.

Nam æquum certè erit has laudes ad *Newtonum* transferre, VIRTUTIS VERÆ cuſtodem, SAPIENTIÆ rigidum ſatellitem, quas de *Epicuro,* "*inſanientis ſapientiæ*" *patre* et "*patrono*" falsò prædicat *Lucretius* ; qui ipſe, vitæ pertæſus, non ultrà quadrageſimum ætatis annum provectus, necem ſibi conſcivit,

MISERÆ PHILOSOPHIÆ MISERUM SOLAMEN !

ANA-

ANALYSIS FLUXIONUM.

PARS SECUNDA.

DEFINITIONES.

§ 78. 1. METHODUS *Fluxionum* eſt quæ determinat quantitatum ejuſdem generis, utcunque variabilium, rationes ad ſe invicèm, quolibet in ſtatu contemporaneo variationis ſeu mutationis, per *methodum primarum et ultimarum rationum* ſeu *limitum*.

2. *Rationes ultimæ* ſunt *limites*, ad quos quantitatum ſine fine decreſcentium *rationes*, 1, *ſemper appropinquant*; et, 2, quas *propiùs aſſequi* poſſunt quàm *pro datâ quâvis differentiâ*; 3, *nunquam verò tranſgredi*; 4, *nec priùs attingere*, quàm quantitates ipſæ diminuuntur in infinitum. Et par eſt natura *rationum primarum*.

3. *Fluens* eſt quantitas ipſa variabilis, ſed tamen finita, quemvis mutationis ſtatum ſubiens, antequam evaneſcat, ſeu in infinitum augeatur.

4. *Genita* eſt ipſius fluentis magnitudo *certo* quodam variationis puncti, ſeu mutationis ſtatu.

5. *Momentum* eſt fluentis augmentum aut decrementum *momentaneum*; id eſt, tempore quam minimo genitum. Eſtque *fluxioni* proportionale.

6. *Fluxio*, ſeu *Functio Analytica*, ſeu *Differentialis*, eſt *menſura quævis rationis ſecundùm quam variatur fluens*, *quolibet in ſtatu mutationis*.

7. Fluentis ipſius *Fluxio* appellatur *prima*, ſeu *primi ordinis*; hujus iterùm *fluxio* (ſi prima fluxio variabilis ſit) *ſecunda*, ſeu *ſecundi ordinis*; (ſi ſecunda quoque ſit variabilis,) hujus denuò *fluxio*, *tertia*, ſeu *tertii ordinis*; atque ita deinceps, donec ultimò perveniatur ad *fluxionem conſtantem*, ſeu mutatione uniformi procedentem.

8. *Me-*

8. *Methodus Fluxionum Directa*, quæ *ex fluentibus datis invenit fluxiones*; *Methodus Inversa*, vicissìm, *ex fluxionibus datis invenit fluentes*.

9. *Æquatio Fluentialis* seu *Integrans*, quæ ex fluentibus solis constat; *Æquatio Fluxionalis* seu *Differentialis*, quæ fluxiones fluentibus immistas continet.

AXIOMATA.

---

79. 1. *Quantitates*, ut et *Quantitatum Rationes*, quæ sunt *constantes* aut *datæ*, nullas sortiuntur fluxiones.

2. Quantitatum vel Rationum in *maximis* aut *minimis* versantium, vel in *limitibus* ipsis, evanescunt fluxiones.

3. *Fluxio summæ* duarum pluriúmve fluentium æquatur *summæ fluxionum* singularum fluentium cum propriis signis; scilicet, affirmativis, si momenta pro incrementis habeantur; negativis, si pro decrementis.

4. Quantitates quæ *constantèr*, seu *æquabilitèr*, seu *certâ* et *determinatâ quâdam ratione* fluunt, fluxiones tantùm *primas* sortiuntur.

5. Fluxio cujusvis fluentis sive puræ, sive cum cognitis quibuscunque quantitatibus immistæ, est eadem. Sic, si A sit quantitas variabilis, et e quantitas data, quantitatis A et $A \pm e$ eadem erit fluxio a.

6. Eadem fluens *plures* sortiatur fluxiones; si modo sint in datâ ad invicem ratione. Sic quantitatis $A^{\frac{m}{n}}$, cujus prima fluxio est $\frac{m}{n} a A$, secunda erit $\frac{m}{n} \cdot \frac{m}{n} - a^2 A^{\frac{m}{n} - 2}$; vel huic proportionalis, $\frac{\frac{m}{n} \cdot \frac{m}{n} - 1}{2} a^2 A^{\frac{m}{n} - 2}$; nempè, in ratione datâ 1 ad $\frac{1}{2}$.

Pauca insupèr, *præparationis* loco, subjungere libet de *variis formis notationis* quæ in calculo sive fluxionali sive differentiali adhiberi solent.

De

De Notatione Fluxionali.

---

80. *Fluentes* primò deſignabat *Newtonus*, in *Lem.* II. *Lib.* II. *Princip.* per literas majuſculas A, B, C, &c. Et harum *momenta*, (ſeu *fluxiones* momentis proportionales), per minuſculas *a*, *b*, *c*, &c.; quæ relationes quantitatum ipſarum reſpectu magnitudinis benè exponunt. Poſteà verò, alteram illam methodum, jam uſitatiorem, excogitavit, deſignandi fluentes per minuſculas z, *y*, *x*, &c. harúmque fluxiones diverſorum ordinum per puncta ſuper-impoſita, hoc modo:

$\dot{z}$, $\dot{y}$, $\dot{x}$ &c - - - *fluxiones primæ*,

$\ddot{z}$, $\ddot{y}$, $\ddot{x}$ &c - - - *fluxiones ſecundæ*,

$\dddot{z}$, $\dddot{y}$, $\dddot{x}$ &c - - - *fluxiones tertiæ*,

$\ddddot{z}$, $\ddddot{y}$, $\ddddot{x}$ &c - - - *fluxiones quartæ*.

81. Secundùm *notationem differentialem*, minùs elegantèr exponuntur *differentiales* uniuſcujuſque gradus,

dx (ſeu *xi*), - *differentialis prima*,

ddx, - - - *differentialis ſecunda*,

dddx, - - - *differentialis tertia*,

ddddx, - - - *differentialis quarta*.

Secundùm notationem quam uſurpat *De La Grange*, ſi curvæ cujuſvis æquatio ſit $y = fx$, erunt valores, *ſubnormalis*, *ſubtangentis* et *radii curvaturæ* yy^{I}, $\frac{y}{y^{\mathrm{I}}}$, et $\frac{(1+y^{\mathrm{I}2})^{\frac{3}{2}}}{y^{\mathrm{II}}}$, reſpectivè, quæ ſecundùm notationem Newtonianam prodeunt $\frac{y\dot{y}}{\dot{x}}$, $\frac{y\dot{x}}{\dot{y}}$, et $\frac{(\dot{x}^2+\dot{y}^2)^{\frac{3}{2}}}{\dot{x}\ddot{y}}$.—*Cenſore* referente.

In *tertiâ editione Princip.* p. 486, deſignat quoque *Newtonus differentias primas* per *b*, 2*b*; *ſecundas*, per *c*, 2*c*; *tertias*, per *d*, 2*d*, &c.

82. Ingenti autèm Matheſeos commodo, introduxit Newtonus in *Algebram Specioſam* notationem *indicum fractorum* et *negativorum*; quarum rationem brevitèr exponere in animo eſt, ne confundantur tyrones notatione inuſitatâ;

F 2 præſertìm,

præfertìm, cùm non defint auctores, qui ufum indicum *fractorum* et *negativorum* pravâ confufione mifcent.

Indices *integri* quantitatum A^2, A^3, A^4, A^m, indicant varias *poteftates* feu dignitates, per *involutionem* refultantes. Indices *fracti* $A^{\frac{1}{2}}$, $A^{\frac{1}{3}}$, $A^{\frac{1}{4}}$, $A^{\frac{1}{n}}$, varias itidem *radices*, per evolutionem; quales nempè referunt *furdæ* quantitates, fecundùm notationem vulgarem, $\sqrt{A}$, $\sqrt[3]{A}$, $\sqrt[4]{A}$, $\sqrt[n]{A}$. Ex utriúfque autem combinatione, emergent $A^{\frac{2}{3}}$, $A^{\frac{3}{4}}$, $A^{\frac{5}{2}}$, $A^{\frac{m}{n}}$, defignantes nimirùm radices per denominatores expofitas, poteftatum per numeratores, $\sqrt[3]{A^2}$, $\sqrt[4]{A^3}$, $\sqrt{A^5}$, $\sqrt[n]{A^m}$. Indices verò *negativi*, A^{-2}, A^{-3}, A^{-m}, $A^{-\frac{2}{3}}$, $A^{-\frac{3}{4}}$, $A^{-\frac{m}{n}}$, indicant *reciprocos* affirmativorum, $\frac{1}{A^2}$, $\frac{1}{A^3}$, $\frac{1}{A^m}$, $\frac{1}{A^{\frac{2}{3}}}$, $\frac{1}{A^{\frac{3}{4}}}$, $\frac{1}{A^{\frac{m}{n}}}$.

Atque hoc pacto, *fractiones tolli*, et *æquationes concinnari* poffunt, tranfponendo quantitates indicibus negativis infignitas, à numeratore in denominatorem; aut vice versâ.

$$\text{Sic } \frac{A^1}{A^2} = \frac{1}{A^2} = A^{-2};\ \text{et } \frac{A^m}{A^n} = \frac{1}{A^{n-m}} = A^{m-n};\ \text{et, } A^{\frac{1}{4}-3} = \frac{1}{A^{3-\frac{1}{4}}}$$

$$= \frac{1}{A^{2+\frac{3}{4}}};\ \text{et, } A^{\frac{m^2}{n^2}-3} = \frac{1}{A^{3-\frac{m^2}{n^2}}},\ \text{et, fic, } \frac{naA^{n-1}}{A^n} \times \frac{1}{A^n} = \frac{na}{A} \times \frac{1}{A^n} =$$

$$\frac{na}{A^{n+1}} = naA^{-n-1}.$$

De Methodo Fluxionum Directâ.

83. Continentur canones, feu methodi hujúfce regulæ, in theoremate quodam generaliffimo, omnes cafus comprehendente:

Lemma II.

" *Momentum genitæ* ($A^m B^n$) *æquatur momentis* (i. e. *fummæ momentorum*, fcilicet ($a + b$) *laterum fingulorum generantium* (A, B) *in eorundem laterum indices dignitatum* (m, n) *et co-efficientia* ($A^{m-1} B^n$, $B^{n-1} A^m$), *continuè ductis*." Seu fluxio $A^m B^n = maA^{m-1} B^n + nbB^{n-1} A^m$.

N. B. Phrafi

N. B. Phrasi Newtonianâ, " *Lateris cujusque generantis co-efficiens, est quantitas quæ oritur, applicando [totam quantitatem] genitam ad hoc latus.*" Sic lateris A *co-efficiens* est $\frac{A^m B^n}{A} = A^{m-1} B^n$; et lateris B, est $\frac{A^m B^n}{B} = B^{n-1} A^m$.

84. Auspicatur *Newtonus*, à casu simplicissimo.

Cas. 1. *Fluxio rectanguli geniti* (AB) *æquatur fluxionibus laterum generantium* (*a* et *b*) *in ipsa latera* (B et A) *continuè ductis*; seu fluxio AB $= aB + bA$.

Rectanguli cujusvis perpetuò fluentis, sint latera, vel æquabilitèr vel pari inæquabilitate crescentia, per datam quandam temporis particulam, in *medio statu* mutationis, A et B. Sumantur horum semi-momenta antecedentia, pro decrementis, $-\frac{1}{2}a$ et $-\frac{1}{2}b$; consequentia autèm, pro incrementis, $+\frac{1}{2}a$ et $+\frac{1}{2}b$. Laterum igitùr variabilium prodibunt valores successivi, eorúmque differentiæ correspondentes à mediis:

$$\begin{array}{llll} \textit{Val. A.} & \textit{Diff.} & \textit{Val. B.} & \textit{Diff.} \\ \left.\begin{array}{l} A - \frac{1}{2}a \\ A \end{array}\right\} & +\frac{1}{2}a, \text{ et} & \left.\begin{array}{l} B - \frac{1}{2}b \\ B \end{array}\right\} & +\frac{1}{2}b. \\ A + \frac{1}{2}a & \left\{\begin{array}{l} +\frac{1}{2}a \\ \hline +a \end{array}\right. & B + \frac{1}{2}b & \left\{\begin{array}{l} +\frac{1}{2}b \\ \hline +b \end{array}\right. . \end{array}$$

Horum valorum primi erunt minimi, et postremi, maximi; sed fluxiones laterum sunt eorum incrementis *totis* simultaneis, *a* et *b*, proportionales respectivè; et proinde jure exponentur per summas differentiarum, $a + b$.

Rectanguli quoque variabilis, prodibunt valores successivi, eorúmque differentiæ, à medio valore, AB:

$$\begin{array}{lll} & \textit{Val.} & \textit{Diff.} \\ \left.\begin{array}{l} (A - \frac{1}{2}a) \times (B - \frac{1}{2}b) = AB - \frac{1}{2}aB - \frac{1}{2}bA + \frac{1}{4}ab \\ A \times B \quad = AB \end{array}\right\} & & + \frac{1}{2}aB + \frac{1}{2}bA - \frac{1}{4}ab \\ (A + \frac{1}{2}a) \times (B + \frac{1}{2}b) = AB + \frac{1}{2}aB + \frac{1}{2}bA + \frac{1}{4}ab & & \left\{\begin{array}{l} + \frac{1}{2}aB + \frac{1}{2}bA + \frac{1}{4}ab \\ \hline + aB + bA \;* \end{array}\right. . \end{array}$$

(*g*) " *Quantitates differentiis suis proportionales sunt continuè proportionales.*" Et conversìm *quantitatum continuè proportionalium erunt differentiæ ipsis quantitatibus proportionales.*

1. Sit A ad A − B ut B ad B − C, et C ad C − D, &c. Et *convertendo* fiet A ad B ut B ad C, et C ad D.

2. Sit A ad B ut B ad C, &c. Et iterùm fiet A ad A − B ut B ad B − C.

Vide *Maclaurin's Fluxions*, § 159—160, ubi prolixè demonstratur et Theoriæ Fluxionum Logarithmorum applicatur.

Et horum valorum quoque, erit primus minimus, et poſtremus, maximus. Sed fluxio rectanguli incrementis laterum *totis* a et b geniti, erit incremento toti proportionalis; et proindè jure exponetur per ſummam differentiarum, $aB + bA$.

Si negas, fingatur fluxionem veram eſſe vel majorem vel minorem, quantitate quâvis aſſignabili D, ſeu, $aB + bA \pm D$. Quod ſi major foret, emergere *poſſit* dimidium novæ fluxionis, $\frac{aB + bA + D}{2}$ majus maximâ, (per conſtructionem) differentiarum, $\frac{1}{2}aB + \frac{1}{2}bA + \frac{1}{4}ab$; et proindè non ritè exprimeret augmentum rectanguli poſt medium ſtatum AB: ſin minor, emergere *poſſit* dimidium novæ fluxionis $\frac{aB + bA - D}{2}$, minus minimâ differentiarum, $+ \frac{1}{2}aB + \frac{1}{2}bA - \frac{1}{4}ab$; et proindè non ritè exprimeret augmentum rectanguli ante medium ſtatum AB. Q. E. D.

In numeris res patebit:

Sit $A = 20$, et $a = 2$; $B = 30$, et $b = 4$; erunt

	Val.	*Diff.*
$(A - \frac{1}{2}a) \times (B - \frac{1}{2}b) = 19 \times 28 =$	532	68
$A \times B$ - - - $= 20 \times 30 =$	600	
$(A + \frac{1}{2}a) \times (B + \frac{1}{2}b) = 21 \times 32 =$	672	72
		140.

Sed $aB + bA = 60 + 80 = (140 =)\ 68 + 72$.

85. Demonſtrationem totam breviſſimè expedit *Newtonus:*

" Rectangulum quodvis *motu perpetuo* auctum AB, *ubi* de lateribus A et B deerant momentorum dimidia $\frac{1}{2}a$ et $\frac{1}{2}b$, fuit $A - \frac{1}{2}a$ in $B - \frac{1}{2}b$, ſeu $AB - \frac{1}{2}aB - \frac{1}{2}bA + \frac{1}{4}ab$; et *quàm primùm* latera A et B alteris momentorum dimidiis aucta ſunt, evadit $A + \frac{1}{2}a$ in $B + \frac{1}{2}b$, ſeu $AB + \frac{1}{2}aB + \frac{1}{2}bA + \frac{1}{4}ab$. De hoc rectangulo ſubducatur rectangulum prius, et manebit *exceſſus* $aB + bA$. —Igitùr laterum incrementis *totis* a et b, generatur rectanguli incrementum $aB + bA$." Q. E. D.

In hâc demonſtratione, *Newtonus*, neglecto *medio* valore rectanguli fluentis, nempè AB, *ſimùl ac ſemèl*, ſubducit minimum valorem de maximo; et inde, *exceſſus* deſignabit totum rectanguli augmentum *totis* momentis a et b generatum, (ſive *ſummam* differentiarum minimi valoris à medio, et medii à maximo); ſed *argumentum ad abſurdum* omittit, quaſi ex ſuis principiis in *Lemmate primo* ſatìs manifeſtum. Vid. § 16. Not. (c).

86. Demonſtrationis *Newtonianæ* vim atque ſubtilitatem minimè capiens " *Analyſta* " *(Berkeley,)* validitatem ejus imperitè improbavit. Nec ſatìs perſpexerunt vindices, *Colſon* et *Maclaurin: Colſon* enìm *argumentum ad abſurdum*, propoſitioni plenè, perfectè et rigidè demonſtrandæ neceſſarium, omiſit; *Maclaurin* verò viciſsìm, priorem demonſtrationis partem, à *Colſon* benè explicatam, paritèr

parièr omisit; huic parti posteriori tantùm immorans, prolixiùs et obscuriùs, in demonstratione parallelâ fluxionis *quadrati*; unde, ordine inventoris lucido in o jus mutato, fluxionem *rectanguli* deducit; *non benè relictâ* interpretis *parmulâ*: sicùt in *Parte Primâ* monuimus, § 16.

87. Analystæ recentiores quoque, *Euler*, (§ 72.) &c. solidam et rigidam demonstrandi viam *Newtonianam* deserentes, rem paulò alitèr instituunt, ex *methodo infinitesimalium*.

Laterum generantium sint postremi valores, $x + \dot{x}$ et $y + \dot{y}$; emerget postremus quoque valor rectanguli simultanei, his in se ductis, $xy + \dot{x}y + x\dot{y} + \dot{x}\dot{y}$. De hoc valore subducatur primus, xy, et manebit *excessus* $\dot{x}y + x\dot{y} + \dot{x}\dot{y}$; nempè incrementis $\dot{x}$ et $\dot{y}$ generatus; cæterùm terminus ultimus $\dot{x}\dot{y}$, utpotè *infinitè parvus*, tutò *negligi* potest: et proindè incrementum ipsius rectanguli quàm proximè exponet $\dot{x}y + x\dot{y}$; sive substitutis, pro incrementis $\dot{x}$ et $\dot{y}$, fluxionibus $\dot{x}$ et $\dot{y}$, fluxionem rectanguli exponet $\dot{x}y + x\dot{y}$. Q. E. D.

Vide *Hutton's Mathematical Dictionary*, voce *Fluxion*, p. 487; undè hanc demonstrationem *Newtonianæ* (ibidem malè omissæ) evidentiâ et elegantiâ longè cedentem, deprompsimus: Nam fluxionem rectanguli variabilis, quam in statu mutationis *primo*, xy, non nisi *quam proximè* exponit $\dot{x}y + x\dot{y}$; in statu *medio*, seu *limite* in ipso, AB, longè accuratiùs exponit aB + bA, secundùm constructionem *Newtonianam* artificiosiorem.

88. Atque ex hujus demonstrationis subtilissimæ genio seu indole, *methodum limitum* indubitatò respiciente, optimè forsàn explicare licet principium illud spinosissimum et vexatissimum, sed nobilissimum et generalissimum sanè, unde pendet omnis theoria *Newtoniana* virium centripetarum, *Princip. Lib.* I. *Prop.* 1. *Cor.* 3. et 4.

Cor. 3. " *Vires centripetæ* in [*mediis* arcuum momentaneorum ABC et DEF] B et E, sunt ad invicèm in *ultimâ ratione diagonalium* BV, EZ, [*quæ convergunt ad centrum virium* S] *ubi arcus isti in infinitum diminuuntur.*"

Nam vires centripetæ, in his mediis locis B et E, utpote *acceleratrices*, propriè mensurantur per *velocitates* versùs commune centrum motûs cadendo acquisitas, æqualibus his temporibus quàm minimis; atque hæ iterùm *velocitates genitæ*, utpote temporum quam minimorum decursu, ferè æquabiles, ob hæc tempora, per hypothesin, æqualia, erunt ut *spatia*, impulsibus vis centripetæ in locis B et E interim generata versùs commune centrum motûs S; sive *deflexiones* de extremis c et f tangentium per B et E ductarum; cæterùm hæ deflexiones *ferè* (b) coincident cum *subtensis* angulorum contactûs cC, fF, *ubi arcus isti*

(b) Vide *La Caille, Elémens d'Astronomie*, seu Versionem Anglicanam à *Robertson*, § 157, et fig. 33. ubi gradus hic demonstrationis præcipuus, à *Newtono* omissus, et à commentatoribus malè neglectus, optimè suppletur: ibi enim notantur *hæ deflexiones veræ* versùs centrum per QR, pF, quæ cum *subtensis* angulorum contactûs, radio vectori parallelis respectivè, *ferè* coincidunt.

ABC et DEF *in infinitum diminuuntur* ; (nimirùm recedendo utrinque ab extremis A et C, atque D et F, versùs media B et E, respectivè) ; itémque hæ subtensæ cC, fF ;—si resolvatur totus revolutionis motus per *semiarcus* BC et EF, (qui utique cum *chordis* suis et cum *tangentibus* suis Bc et Ef ultimò coincident, per *Lem.* 7.) binos in motus, unum, secundùm directiones tangentium, et alterum, secundùm directiones radiorum vectorum BS, et ES,—erunt parallelæ et æquales *diagonalibus* BV et EZ, quæ convergunt ad centrum virium et chordas AC, DF bisecant : ergò *vires*, &c. erunt *ipsis diagonalibus* proportionales. Q. E. D.

In hâc demonstratione, (quam, brevissimè à *Newtono* absolutam, fusiùs explicuimus,) quis non videt quod "*ubi arcus isti in infinitum diminuuntur*," nimirùm recedendo utrinque, &c. semiarcus priores AB, DE antè media puncta B et E, *decrementis* $-\frac{1}{2}a$, $-\frac{1}{2}b$; et semiarcus posteriores BC, EF, *incrementis* $+\frac{1}{2}a$, $+\frac{1}{2}b$, respondent respectivè ?

89. *Cor.* 4. "*Vires illæ centripetæ*, &c. sunt quoque proportionales *sagittis illis* quæ chordas bisecant arcuum quorumvis, *æqualibus* temporibus quam minimis descriptorum, et convergunt ad centrum virium."

Nam hæ sagittæ sunt *semisses diagonalium* prædictarum, seu $\frac{1}{2}$BV, et $\frac{1}{2}$EZ ; et proinde pro ipsis diagonalibus, (*veris* utique virium exponentibus,) compendii gratiâ, calculo cautè instituto, et servatâ ubique *semissium proportione*, per totum computum, tutò substitui possunt. Alitèr enim suboriri possunt errores in calculo minimè contemnendi. Vide *Maclaurin's Fluxions*, § 440 ; *Simson's Fluxions*, Vol. I. p. 237 ; *La Lande, Astronomie*, § 3392 ; et *Milner*, *Philosoph. Transf.* anno 1779, *Part* II. p. 521.—Vide plura, § 103, et anteà § 62 et 63.

90. Quò magìs eluceant evidentia, certitudo et ubertas *primi hujus casûs*, quem totius methodi generalis fundamentalem constituit *Newtonus*, placet insupèr ex considerationibus *Geometricis*, abstractiùs et subtiliùs hactenùs tradita, illustrare.

1. Lateribus generantibus, A et B, *uniformitèr* crescentibus, sive in eâdem, sive in datâ quâdam ratione ad se invicèm ; erit *diagonalis* rectanguli duplici motu geniti *recta linea*. Et *complementa* parallelogrammorum circà diagonalem ritè exponent fluxiones laterum respectivè ; adeóque horum aggregatum fluxionem ipsius rectanguli geniti. Sed in hoc casu simplicissimo, erunt ipsa complementa æqualia, per *Prop.* 43. *Lib.* I. *Elem.* ; sive $aB = bA$, adeóque areæ rectanguli fluxio $= 2aB$ vel $2aA$. Et si latera sint æqualia, erit fluxio *quadrati* $= 2aA$.

2. Lateribus *difformitèr* crescentibus, seu in diversâ ab invicèm ratione ; ex. gr. si momentum a sit constans, at momentum b variabile, erit *diagonalis* rectanguli geniti *curva linea* : quæ erit *convexa* à parte baseos seu abscissæ motu constanti descriptæ, si b crescat ; *concava* autèm, si decrescat. In utróque casu, complementa illa quæ fluxiones laterum exponunt, erunt inæqualia.

Hinc forsàn *Sectiones Conicæ*, nomina sua propria acceperunt: *Hyperbola*, quasi motu ordinatæ ὑπερβαλλοντι, "*redundante*," genita; *Ellipsis*, quasi motu ελλειποντι, "*deficiente*;" et *Parabola*, παραβαλλοντι, seu motum ad *constantem* seu æquabilem *accedente*, genita. Atque his definitionibus nominum affectiones singularum sectionum probè respondent.

3. Crescente uno latere et decrescente altero, sive *constantèr*, sive *diversimodè*, ob momentum posterioris jam negativum, $-b$, rectanguli fluxio fiet residua, $aB - bA$.

Casu hoc primo, omnium fundamentali, jam rigidè demonstrato ex principiis *purè Algebräicis*, omni *motûs formalis* seclusâ consideratione; et fusiùs illustrato ex *Mechanicâ Rationali* atque *Geometriâ*, secundùm mentem ipsius inventoris; reliquos casus breviùs expedire fas erit; quibusdam aliundè intertextis.

91. *Cas.* 1. *Cor.*—*Hinc fluxio fractionis* $\frac{A}{B}$ *fiet* $\frac{aB - bA}{B^2}$. Ponatur $\frac{A}{B} =$ Q: eritque $A = BQ$. Adeóque per *Cas.* 1. $a = qB + bQ$; sive $q = \frac{a}{B} - \frac{bQ}{B} = \frac{a}{B} - \frac{b}{B} \times Q = \frac{a}{B} - \frac{b}{B} \times \frac{A}{B} = \frac{a}{B} - \frac{bA}{B^2} = \frac{aB}{B^2} - \frac{bA}{B^2} = \frac{aB - bA}{B^2}$. Q. E. D.

92. *Fluxio contenti sub lateribus quotcunque*, A, B, C, &c. *æquatur summæ ex fluxionibus singulorum in reliqua latera sigillatim continuè ductis.* Sic fluxio ABC, &c. $= a$BC, &c. $+ b$AC, &c. $+ c$AB, &c.

Ponatur $AB = G$. Et per *Cas.* 1. erit fluxio ABC, seu $GC = gC + cG = (aB + bA) \times C + c \times AB = aBC + bAC + cAB$. Et par est ratio contenti sub lateribus quotcunque.

93. *Cas.* 3. *Fluxio dignitatis cujusvis affirmativæ* (A^m) *æquatur indici illius dignitatis* (m), *fluxioni lateris* (a), *ejúsque co-efficienti* (A^{m-1}), *in se continuè ductis.* Seu, fluxio $A^m = maA^{m-1}$.

Fluxio quadrati A^2, seu $A \times A$, erit $aA + aA = 2aA$, per *Cas.* 1. Fluxio itèm cubi A^3, seu $A \times A \times A$, erit quoque $aA^2 + aA^2 + aA^2 = 3aA^2$, per *Cas.* 2. Et, eodem argumento, fluxio $A^m = maA^{m-1}$. Q. E. D.

Cor. Hinc fluxio $\frac{A^m}{m}$ erit aA^{m-1}.

Scholium. Proprietate curvarum elegantiſſimâ, ſi area quævis curvilinea ſuper *baſi* (AP) deſignetur per A^m, quolibet in ſtatu mutationis; ſemper exponetur *ordinata* (PM) per maA^{m-1}. Vide *Maclaurin*'s *Fluxions*, § 936, fig. 352.

94. *Caſ.* 4. *Fluxio dignitatis cujuſvis negativæ* (A^{-m}) *æquatur indici, in lateris fluxionem ductæ* ($-ma$), *et per indicem genitæ unitate auctum diviſæ* ($m+1$). Seu fluxio $A^{-m} = \frac{-ma}{A^{m+1}}$.

Quantitatis conſtantis $A \times \frac{1}{A} = 1$, erit fluxio nihil, per *Ax.* 1. Fiat $\frac{1}{A} = B$; erítque fluxio $AB = aB + bA = 0$. Adeóque b, ſeu fluxio $\frac{1}{A}$ vel $A^{-1}) = \frac{-aB}{A} = \frac{-a}{A} \times \frac{1}{A} = \frac{-a}{A^2}$. Et generalitèr, quantitatis conſtantis $A^m \times \frac{1}{A^m} = 1$, fluxio erit nihil. Fiat $\frac{1}{A^m} = B$, erítque fluxio $A^m \times B = bA^m + maBA^{m-1} = 0$. Adeóque b, (ſeu fluxio $\frac{1}{A^m}$ vel A^{-m}) $= \frac{-maBA^{m-1}}{A^m} = \frac{-maB}{A^1} = \frac{-ma}{A^1} \times \frac{1}{A^m} = \frac{-ma}{A^{m+1}}$. Q. E. D.

95. *Caſ.* 5. *Fluxio cujuſcunque dimenſionis fractæ affirmativæ* ($A^{\frac{m}{n}}$ *æquatur indici illius dimenſionis* ($\frac{m}{n}$), *lateris fluxioni* (a), *ejúſque co-efficienti* ($A^{\frac{m}{n}-1}$), *in ſe continuè ductis.* Seu fluxio $A^{\frac{m}{n}} = \frac{m}{n} aA^{\frac{m}{n}-1}$.

Quantitatis $A^{\frac{1}{2}} \times A^{\frac{1}{2}} = A$ fluxio erit a. Fiat $A^{\frac{1}{2}} = B$; adeóque fluxio B^2 (ſeu $A^{\frac{1}{2}} \times A^{\frac{1}{2}}$) erit $2bB = a$; et proinde b (ſeu fluxio $A^{\frac{1}{2}}$) $= \frac{a}{2B} = \frac{a}{2A^{\frac{1}{2}}}$. Et generalitèr: fiat $A^{\frac{m}{n}} = B$; erítque $A^m = B^n$; et proinde harum fluxiones erunt inter ſe æquales, nempè $maA^{m-1} = nbB^{n-1}$; et di-

 videndo

videndo utrinque per $A^m = B^n$, erit $maA^{-1} = nbB^{-1}$; ideóque erit b (seu fluxio B, sive $A^{\frac{m}{n}}$) $= \frac{maA^{-1}}{nB^{-1}} = \frac{maA^{-1}}{n \times A^{-\frac{m}{n}}} = \frac{maA^{\frac{m}{n}-1}}{n}$. Q. E. D.

96. N. B. Pro co-efficientis indice, $\frac{m}{n} - 1$, usurpat *Newtonus* indicem æquipollentem $\frac{m-n}{n}$ $(= \frac{m}{n} - \frac{n}{n}) = \frac{m}{n} - 1$, in demonstratione hujus casûs; unde prodit fluxio $A^{\frac{m}{n}} = \frac{m}{n} aA^{\frac{m-n}{n}}$. Eâdem nimirùm notatione indicum hâc occasione usus, ac compluribus ante annis, nempè anno 1676, ubi primò præclarum suum *Theorema Binomium* in lucem protulit. Vid. ANALYS. ÆQUAT. § 57. Cæterùm hoc non levem suppeditat conjecturam, Methodum Fluxionum, vel illo tempore, Newtono perspicuè innotuisse; ut posthàc pleniùs et irrefragabiliùs evincetur ex insigni analogiâ inter terminos successivos seriei binomiæ, atque fluxiones superiorum ordinum, intercedente; talem enim analogiam inter utramque methodum, utriusque inventorem effugisse, suspicari non fas est.

97. *Cas.* 6. *Fluxio genitæ cujuscunque ex duobus pluribúsve factoribus, æquatur summæ ex fluxionibus uniuscujusque factoris in reliquam factorem vel reliquos factores, continuè inter se ductis.* Sic fluxio $A^m B^n = maB^n A^{m-1} + nbA^m B^{n-1}$.

Ex præcedentibus constat; nempe, *Cas.* 1 et 3. Et par est ratio contenti sub pluribus factoribus; nam, per *Cas.* 2, erit fluxio $A^m B^n C^r = maB^n C^r A^{m-1} + nbA^m C^r B^{n-1} + rcA^m B^n C^{r-1}$.

Idem quoque obtinebit per *Cas.* 5, ubi indices factorum sunt *fracti*, $\frac{m}{n}$, $\frac{r}{s}$, &c. $A^{\frac{m}{n}} B^{\frac{r}{s}}$; cujus fluxio evadet $\frac{m}{n} aB^{\frac{r}{s}} A^{\frac{m}{n}-1}$ + &c.

Et hoc est *Theorema Fluxionale* nobilissimum et generalissimum, uno intuitu omnes casus comprehendens, ipsi *Theoremati Binomio*, elegantiâ, concinnitate et dignitatis præstantiâ vix, et ne vix quidem, cedens.

98. Conclusiones præcedentes nullo negotio ad notationem posteriorem accommodare licet.

Fluxio *rectanguli* - $xy = \dot{x}y + x\dot{y}$.
Fluxio *contenti* - $xyz = \dot{x}yz + x\dot{y}z + xy\dot{z}$.
Fluxio *quoti* - $\frac{x}{y} = \frac{\dot{x}y - x\dot{y}}{y^2}$
Fluxio *quadrati* - $x^2 = 2\dot{x}x$.
Fluxio *dignitatis* $x^m = m\dot{x}x^{m-1}$.
Fluxio *dimensionis* $x^{\frac{m}{n}} = \frac{m}{n}\dot{x}x^{\frac{m}{n}-1}$
Fluxio *genitæ* - $x^m y^n = m\dot{x}x^{m-1}y^n + n\dot{y}y^{n-1}x^m$, vel,
$= (m\dot{x}y + nx\dot{y})\,x^{m-1}y^{n-1}$, vel,
$= \left(\frac{m\dot{x}}{x} + \frac{n\dot{y}}{y}\right)x^m y^n$.

—Itémque, $x^m y^n z^r = m\dot{x}x^{m-1}y^n z^r + n\dot{y}y^{n-1}x^m z^r + r\dot{z}z^{r-1}x^m y^n$,
vel, $= (m\dot{x}yz + nx\dot{y}z + rxy\dot{z})\,x^{m-1}y^{n-1}z^{r-1}$,
vel, $= \left(\frac{m\dot{x}}{x} + \frac{n\dot{y}}{y} + \frac{r\dot{z}}{z}\right)x^m y^n z^r$.

99. Corollaria quædam *Newtoniana*, à *Maclaurin* omiſſa, hìc adjungere libet, ex præcedente theoriâ reſultantia, eandémque optimè illuſtrantia, et in *Logarithmotechniâ* valdè utilia.

Corol. 1. *Hinc, in continuè proportionalibus, ſi terminus unus datur, fluxiones terminorum reliquorum, erunt ut iidem termini multiplicati per numerum intervallorum inter ipſos et terminum datum.*

Sunto A, B, C, D, E, F, continuè proportionales, et ſi *detur* terminus intermedius C, (cujus idcircò fluxio erit *nihil*,) exquirantur tùm valores reliquorum, ex notis proportionalium proprietatibus, tùm horum fluxiones, ex præcedenti theoriâ, itémque fluxionum rationes:

Proport.	*Valor.*	*Flux.*	1 *Rat.*	2 *Rat.*
A ſive	$\frac{C^3}{D^2}$	$C^3 \times \frac{-2d}{D^3}$	$-\frac{2C^3}{D^2}$	-2A
B	$\frac{C^2}{D}$	$C^2 \times \frac{-d}{D^2}$	$-\frac{C^2}{D}$	-1B
C	1	0	0	0
D	D	d	$+$D	$+1$D
E	$\frac{D^2}{C}$	$\frac{2dD}{C}$	$+\frac{2D^2}{C}$	$+2$E
F	$\frac{D^3}{C^2}$	$\frac{3dD^2}{C^2}$	$+\frac{3D^3}{C^2}$	$+3$F.

In

In hâc tabulâ, ex ferie fluxionum elicitur prima feries rationum $-\frac{2C^3}{D^2}$, $-\frac{C^3}{D}$, &c.; multiplicando nimirùm fingulas fluxiones correfpondentes, per communem quantitatem $\frac{D}{d}$; fecunda feries autèm, ex primâ, reftituendo pro valoribus fuis, proportionales ipfos, $-\frac{2C^3}{D^2} = -2A$, &c. Q. E. D.

100. *Corol.* 2. *Et fi in quatuor proportionalibus duæ mediæ dentur, fluxiones extremarum erunt ut eædem extremæ.*

Nam, ob datum idcircò rectangulum fub extremis AD (utpote æquale BC), erit fluxio ejus $aD + dA = 0$; ideóque, $aD = -dA$; et proinde, $a : -d :: A : D$. Q. E. D.

Idem intelligendum eft de lateribus rectanguli cujufcunque dati. Pari enim ratione, erit $b : -C :: B : C$. Cæterùm in hujufmodi cafibus, ob fluxiones negativas $-c$, $-d$, diverfimodè variantur latera generantia, uno crefcente, altero decrefcente.

101. *Corol.* 3. *Et fi fumma vel differentia duorum quadratorum detur, fluxiones laterum erunt reciprocè ut latera.*

Sint enim quadrata $A^2 \pm B^2 = 1$, feu cuivis quantitati datæ; erit fluxio $2aA \pm 2bB = 0$. Adeóque, $2aA = \mp 2bB$; et proinde, $a : \mp b :: B : A$. Q. E. D.

De Fluxionibus Superiorum Ordinum.

---

102. Si quantitates fluentes ratione conftanti, æquabili feu datâ, fimùl augentur vel minuuntur, *fluxiones primas* tantùm, feu *primi ordinis*, fortientur. Quòd fi harum fluxiones fint quoque variabiles, fuas iterùm fluxiones fortientur, *fecundas* nempè, feu *fecundi ordinis*; atque hæ iterùm fuas fortientur fluxiones feu *tertias*; atque itâ deinceps, donec ultimò perveniatur ad quantitates conftantèr feu æquabilitèr fluentes.

103. Sic in theoriâ *virium centripetarum*, binis viribus conjunctis urgentibus, revolvuntur corpora in orbibus, feu trajectoriis quibufcunque, circà commune centrum virium: nempè, *vi projectili*, quæ conftantèr et æquabilitèr agit, fecundùm directiones tangentium fingulis in orbitæ locis; et *vi centripetâ*, perpetuo motu *variabili*, corpora de tangentibus versùs centrum deflectente; cùm actio

vis

vis projectilis igitùr est *constans per se* (nisi quatenùs per alteram vim *obliquè* agentem, augetur vel minuitur), *primas* tantùm pariet fluxiones; quarum exponentes erunt *spatia* perbrevia æqualibus temporibus quàm minimis percursa, seu *arcus* momentanei cum *tangentibus* suis ferè coincidentes. At *vis centripetæ*, cum accedendo versùs centrum, fortiùs, recedendo à centro, languidiùs operatur, actio erit variabilis; ejúsque exponentes in locis correspondentibus, non erunt *ipsa spatia*, *momentanea* cadendo versùs centrum percursa, si sisteretur vis projectilis (cum ipsæ deflexiones erunt variabiles, pro diversis à centro distantiis), sed *horum dupla*, seu spatia quæ velocitatibus *ultimis* cadendo acquisitis describerentur (si exindè constantèr velocitates forent uniformes), per eadem tempora quam minima. *Vis centripeta*, igitùr, *fluxiones* suas *secundas* sortietur; quarum exponentes sunt *diagonales illæ*, suprà memoratæ, quæ ad centrum commune virium convergunt: hæ enim sunt *duplæ sagittarum* illarum, quas corpora his momentis liberè cadendo, *actu* describerent. In *circulis* autèm hæ sagittæ sunt ipsi *sinus versi*.—Atque hæ observationes, theoriæ illæ difficillimæ ac subtilissimæ virium *projectilium* et *centripetarum*, cui innititur omnis ASTRONOMIA NEWTONIANA, explicandæ et illustrandæ, haud leve adjumentum forsàn tyronibus suppeditabunt.

104. *Fluxiones superiorum ordinum* iisdem legibus inveniendæ sunt ac ipsæ *fluxiones primæ*. Sic generalitèr: genitæ cujuscunque $A^{\frac{m}{n}}$ erit

Fluxio prima, per *Cas.* 5, - - - - $\frac{m}{n} a A^{\frac{m}{n}-1}$;

Secunda, — $(\frac{m}{n}-1) \times (a) \times (\frac{m}{n} a A^{\frac{m}{n}-2} = \frac{m}{n} \cdot \frac{m}{n} - 1\, a^2 A^{\frac{m}{n}-2}$;

Tertia, — $(\frac{m}{n}-2) \times (a) \times (\frac{m}{n} \cdot \frac{m}{n} - 1\, a^2 A^{\frac{m}{n}-3}) = \frac{m}{n} \cdot \frac{m}{n} - 1 \cdot \frac{m}{n} - 2a^3 A^{\frac{m}{n}-3}$.

Et sic deinceps, eâdem servatâ lege derivationis. Ut, si cubi A^3 quærantur fluxiones: designante $\frac{m}{n}$, 3; erit *fluxio prima*, - - $3aA^2$;

secunda, - $6a^2A$;

tertia, - - $6a^3$.

Fluxio autèm *quarta* erit *nihil*; jam evanescente factore A.

Hinc, si genitæ index sit numerus integer, series erit finita; sin fractus, indefinita.

105. Com-

105. Combinando autèm terminos inter operandum inventos, breviùs exponere licet ſeriem fluxionalem, ponendo nimirùm, $\frac{m}{n} = p$; $\frac{m}{n} \cdot \frac{m}{n} - 1 = q$; $q \times \frac{m}{n} - 2 = r$; erit $paA^{\frac{m}{n}-1}$, *fluxio prima*,

$qa^2A^{\frac{m}{n}-2}$, *fluxio ſecunda*,

$ra^3A^{\frac{m}{n}-3}$, *fluxio tertia*,

$sa^4A^{\frac{m}{n}-4}$, *fluxio quarta.*

Atque ulteriùs combinando, formam adhuc ſimpliciorem indicet series; ponendo $pa = P$; $\overline{\frac{m}{n} - 1} \times aP = Q$; $\overline{\frac{m}{n} - 2} \times aQ = R$. $\overline{\frac{m}{n} - 3} \times aR = S$, &c. erit jam ſeries,

$PA^{\frac{m}{n}-1} + QA^{\frac{m}{n}-2} + RA^{\frac{m}{n}-3} + SA^{\frac{m}{n}-4}$ + &c; ipſi *ſeriei binomiæ*, in ſimplicitate non inferior.

De Theoremate Binomio.

---

106. In *Appendice* ad ANALYSIN ÆQUATIONUM, p. 33, &c. dudùm demonſtratum eſt, "*ex principiis Algebræ propriis*," (ut ibi dictum,) ad mentem *Johannis Landen*, in tractatu ſuo *De Analyſi Reſiduâ*, theorema hoc longè celeberrimum, ſub variis ejus formulis, § 57; (quales protulit *Newtonus*, in Epiſtolis cum *Leibnitz* communicandis, an. 1676,) atque ſub formulâ uſitatiore, *Cor.* 1. qualem non ſemèl adhibuit ipſe *Newtonus*. Ex. gr. *Prop.* 45. *Lib.* 1. *Princip.* ubi ſtatuit *radium vectorem* $A^n = (T - X)^n = T^n - nXT^{n-1} + \frac{n^2 - n}{2}X^2 T^{n-2}$, &c. Atque iterùm, in *Scholio*, *Prop*, 93, *ordinatam* $(A + O)^{\frac{m}{n}} = A^{\frac{m}{n}} + \frac{m}{n} O A^{\frac{m-n}{n}} + \frac{mm - mn}{2nn} O^2 A^{\frac{m-2n}{n}}$, &c.—Pleniorem et fuſiorem expli-

cationem

cationem Methodi *Landenianæ* inveniet lector horum studiosus, apud *Maseres*, in libro cui titulus *Scriptores Logarithmici*, Vol. II. p. 170, &c. et 176, &c.

107. Verùm hanc methodum perpendenti, atque iterùm ad examen revocanti, vix affirmare licet, ex principiis *merè Algebräicis* eam prorsùs peti: assumptio enim cui potissimùm innititur demonstratio, nempè, " quòd *salvâ æquatione variari potest y ad libitum*, donec fiat æqualis ipsi x in singulis dimensionibus fractionis $\frac{1+y}{1+x}$," calculum *fluxionalem* planè redolet, qualem in demonstratione *tertii* casûs, § 93, antecedentis jam perspeximus, æquatis valoribus A, B, C, &c.

108. Admirandam sanè intercedere analogiam inter *Theorema Binomium* et *Theoriam Fluxionum*, liquidò constat ex collatione terminorum seriei resultantis, cum diversis fluxionum ordinibus suprà expositis, § 105.

Exponat enim A quantitatem quamvis generantem, et a hujus *momentum*, seu fluxionem, sive crescendi sive decrescendi; designabit igitùr $A \pm a$ valorem *generantis*, et $(A \pm a)^{\frac{m}{n}}$ *genitæ*, simùl ac semèl fluentium. Hujus autem resolutio dabit seriem, citissimè convergentem, cui apponitur series quoque fluxionum diversorum ordinum.

	Series Binomia.	*Series Fluxionum.*	
1.	$A^{\frac{m}{n}}$ - - - - - -	$A^{\frac{m}{n}}$ - - - - - -	*Fluens.*
2.	$\pm \frac{m}{n} a A^{\frac{m}{n}-1}$ - -	$\frac{m}{n} a A^{\frac{m}{n}-1}$ - - - -	1 *Fluxio,*
3.	$\frac{m}{n} \cdot \frac{\frac{m}{n}-1}{2} \times a^2 A^{\frac{m}{n}-2}$ -	$\frac{m}{n} \cdot \frac{m}{n}-1\, a^2 A^{\frac{m}{n}-2}$ -	2 *Fluxio,*
4.	$\pm \frac{m}{n} \cdot \frac{\frac{m}{n}-1}{2} \cdot \frac{\frac{m}{n}-2}{3} \times a^3 A^{\frac{m}{n}-3}$	$\frac{m}{n} \cdot \frac{m}{n}-1 \cdot \frac{m}{n}-2\, a^3 A^{\frac{m}{n}-3}$	3 *Fluxio.*
	&c.	&c.	

Hinc constat, quòd *primus* seriei terminus est *ipsa fluens*; *secundus* terminus, *ipsa prima* fluxio; reliqui autem terminis fluxionibus superiorum ordinum, *proportionales* respectivè, et proinde pro his usurpari possunt; per fluxionum definitionem, " mensuræ *quævis* rationum," &c.

Nam

Nam tertius terminus est ad secundam fluxionem in ratione $\frac{1}{2}$ ad 1, seu 1 ad 2.

Quartus terminus ad tertiam fluxionem in ratione $\frac{1}{2} \times \frac{1}{3}$ ad 1, seu 1 ad $2 \times 3 = 6$.

Quintus terminus ad quartam fluxionem in ratione $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$ ad 1, seu 1 ad $2 \times 3 \times 4 = 24$.

Et generalitèr, terminus quilibet $m + 1$, ad fluxionem ordinis m proximè inferioris, in ratione $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \text{ - - - } \frac{1}{m}$ ad 1, seu 1 ad $2 \times 3 \times 4 \times 5$ &c $\times m$; hoc est, in ratione datâ.

109. Et hinc facillimè quidèm, et optimè forsàn, ex theoriâ fluxionum *germanâ* derivatur constructio seriei ex resolutione theorematis binomii resultantis; nam quilibet seriei terminus $m + 1$ invenietur dividendo fluxionem ordinis m per $2 \times 3 \times 4 \text{ - - - } m$. Sic terminus seriei *sextus* est æqualis fluxioni *quintæ*, $t a^5 A^{\frac{m}{n} - 5}$, per 24×5 divisæ. Et vicissìm, ex serie binomiâ inveniri possunt fluxionum ordines.

110. Simplicitate atque facilitate longè præstat hæc demonstratio, ex fluxionum *serie* petita, *Landenianæ* isti breviori, ex consideratione *primæ* fluxionis tantùm haustæ; ab eâ quam dudùm exposuit *Maclaurin*, § 748, vix discrepanti; quippe quæ primo *assumit*, quantitatem binomiam $(1+x)^{\frac{m}{n}}$ designari posse per seriem indicibus integris, perpetuò crescentem, $1 + ax + bx^2 + cx^3 + dx^4$, &c. Et harum quantitatum fluxionibus utrinque inventis, erit $\frac{m}{n} \times (1 + x)^{\frac{m}{n} - 1} \times \dot{x} = a\dot{x} + 2bx\dot{x} + 3cx^2\dot{x} + 4dx^3\dot{x}$, &c. unde, dividendo omnes terminos per $\dot{x}$, emergit æquatio finalis $\frac{m}{n} \times (1 + x)^{\frac{m}{n} - 1} = a + 2bx + 3cx^2 + 4dx^4$, &c.; eadem prorsùs ad quam tanto circuitu tandem pervenit methodus *Landeniana*; vide *Analysin Æquationum*, p. 38. Cujus iterùm resolutio, càlculo satìs operoso, dabit valores singulorum co-efficientium, b, c, d, &c.

Hinc manifeſtò liquet, *Analyſin Reſiduam* ipſam, cujus laudes tantopere prædicat *Landen*, vix diſcrepare in principiis ſuis, licet longè operoſiorem ac tortuoſiorem, à *Methodo Fluxionum*; ac proinde pro larvâ ejus jure cenſeri poſſe.

111. Ex dictis conſtat, *ſeriem binomiam* $(A \pm c)^{\frac{m}{n}}$ citiùs convergere quam *ſeries fluxionum genitæ* $A^{\frac{m}{n}}$; ob decrementum terminorum prioris per denominatores ſucceſſivos terminis poſterioris ſubjiciendos, $1 \times 2 = 2$; $2 \times 3 = 6$; $6 \times 4 = 24$; &c.

Sic, $(A + a)^3 = A^3 + 3aA^2 + 3a^2A + a^3$.

Fluxiones, A^3, $= 3aA^2 + 6a^2A + 6a^3$.

De Methodo Inverſâ Fluxionum.

112. *Methodus Inverſa*, ſeu *Calculus Integralis*, viciſsìm, *ex fluxionibus datis invenire fluentes* aggreditur. Eſtque "*hoc moleſtiſſimum et omnium difficillimum problema*," *Newtono* judice: nam hactenùs deſideratur methodus generalis eruendi fluentes directè, ſeu immediatè ex fluxionibus; ad particulares caſus tantùm reſtringitur inveſtigatio, eoſdémque, maximâ ex parte, curſum relegendo methodi directæ.

Auſpicari libet à caſibus ſimplicioribus.

113. *Caſ.* 1. *Fluxionum ſimplicium unius tantùm dimenſionis fluentes ſubſtitutione ſolâ inveniuntur.*

Sic fluxionis $\dot{a}$, fluens eſt A; fluxionis $\frac{m}{n}\dot{a}$, $\frac{m}{n}$A; vel fluxionis $\dot{x}$, fluens x; $\frac{m}{n}\dot{x}$, $\frac{m}{n}x$.

114. *Caſ.* 2. *Fluxionum ſimplicium plurium dimenſionum, fluentes reſolutione regularum methodi directæ inveniuntur.*

Sic quoniam fluentis A^3 fluxio fuit $3aA^2$; dividendo priorem per poſteriorem, prodit $\frac{A}{3a}$; quæ utique, multiplicando datam fluxionem, dabit fluentem viciſsìm.

cissim. Itémque fluxionis $3\dot{x}x^5$, multiplicando per $\frac{x}{6\dot{x}}$, fluens erit $\frac{3x^6}{6} = \frac{1}{2}x^6$. Et generalitèr, formulæ $\frac{m}{n} a A^{\frac{m}{n} - 1}$, multiplicando per $\frac{A}{\frac{m}{n} a}$, eruetur fluens quæsita $A^{\frac{m}{n}}$. Similitèr, formæ $a A^{m-1}$ erit fluens $\frac{A^m}{m}$, ut ex casibus 3 et 5 methodi directæ constat.

115. Methodum directam excogitavit *Pacassi* inveniendi fluentes, instituendo analogiam: *Ex fluxione* ($\dot{x}$) *datâ, eliciatur nova quædam fluens* (y); *erítque hujus fluxio* ($\dot{y}$) *ad fluxionem datam* ($\dot{x}$) *ut fluens elicita* (y) *ad fluentem quæsitam* (x): et indè prodit $x = \frac{\dot{x}y}{\dot{y}}$.

Sit fluxio data $3aA^2$; substituendo A pro a, emergit nova fluens $3A^3$; inveniatur hujus fluxio, $9aA^2$; undè prodibit $\frac{3aA^2 \times 3A^3}{9aA^2} = A^3$, fluens quæsita. Atque hæc regula, licet præcedente operosior, in *fluentium correctione* per methodum inversam inventarum, usui esse potest.

116. *Fluxionis compositæ, unius tantùm dimensionis formulæ* $\dot{x}y \pm x\dot{y}$, *invenietur fluens, substituendo fluentes pro fluxionibus, et dimidiando aggregatum*; nam $\frac{xy + xy}{2} = xy$. Et paritèr, formulæ $\frac{\dot{x}y - x\dot{y}}{y^2}$ fluens erit $\frac{xy + xy}{2y^2} = \frac{x}{y}$.

117. Veruntamen sæpenumerò occurrunt quantitates fluxionales magìs compositæ, quarum fluentes per regulas prædictas neutiquàm eruere licet. Decursu autem computorum fluxionalium, complures *formulæ fluentiales*, formulis *fluxionalibus* magìs compositis respondentes, inventæ sunt ab analystis celeberrimis, inter operandum; et potissimùm, à *Newtono*, *Cotes*, *Euler*, *Clairaut*, *Walmsley*, *Landen*, *Simpson*, *Waring*, *De La Grange*, &c.; quarum præcipuæ ad calcem capitis *de Fluxionibus Logarithmorum* rejiciuntur, § 132.

De Correctione Fluentium.

118. Cæterùm fluentes hoc pacto repertæ, sive per regulas præcedentes, sive per formulas, haùd raró correctione quâdam ulteriore egent, quo ad

 amussim

amuſſim reſpondeant caſibus problematum fluxionalium particularibus inveniendis. Nam fluxionis $\dot{x}$ fluens eſſe poteſt vel x, vel $x \pm c$, ſeu x conſtanti quâdam quantitate, c, aucta vel diminuta; adeò ut fluens jam inventa, conditionibus quibuſdam, quæ problema aliquatenùs reſtringunt, poterit forsàn non accuratè reſpondere. Unde neceſſe erit ulteriùs fluentem veram eruere ex ipſius problematis conditionibus, relationem fluentium ad invicèm in certo quodam temporis puncto, aut mutationis ſtatu, definientis ſeu limitantis. Reducatur igitùr generalis æquatio fluentialis ad iſtum ſtatum particularem, ſubſtituendo pro fluentibus ipſis cognitos, quos ſortiuntur, valores. Deinde ſubducatur æquatio reſultans ex æquatione generali, et reſidua utrinque æquationem correctam fluentium ex regulis inventarum, elicient.

Ut, ſi æquatio fluxionalis ſit $\dot{z} = ax^3\dot{x}$, æquatio fluentialis poteſt eſſe vel $z = ax^4$, vel $z = ax^4 \pm c$. Si ſimùl evaneſcunt x et z, ſubſtituendo o pro x et z, fiet æquatio $0 = 0 + c$. Adeóque $c = 0$: quo in caſu, ax^4 veram fluentem exponet, evaneſcente c.

2. At, ſi, evaneſcente z, reſtat $x = b$, finitæ cuidam quantitati cognitæ; fiet æquatio fluentialis, in iſto caſu, $0 = ab^4 + c$: unde prodit $c = -ab^4$; et ſubſtituto hoc valore, prodit $z = ax^4 - ab^4$.

3. Fiat jam $z = d$, iſto in ſtatu ubi x fit æqualis b; et his valoribus ſubſtitutis, emerget æquatio fluentialis $d = ab^4 + c$; adeóque erit $c = d - ab^4$; et, ſubſtituto iterùm hoc valore pro c, prodit $z = ax^4 - ab^4 + d$, valor nimirùm correctus fluentis quæſitæ.

4. Idem quoque breviùs expedire licet, ſimùl et ſemèl ſubſtituendo valores quoſvis contemporaneos quantitatum x et z. Nam ſi ab æquatione fluentiali $z = ax^4$, ſubducatur novus valor $d = ab^4$, fiet reſiduum $z - d = ax^4 - ab^4$; adeóque $z = ax^4 - ab^4 + d$, ut anteà: aut igitùr evaneſcentibus b et d, aut æquatis $+d$ et $-ab^4$, fiet ipſa fluens quæſita ax^4.

De Fluxionibus Diverſas Fluentes admittentibus.

---

119. Nullâ re magìs arguitur imperfectio *Methodi Inverſæ*, quam quòd eadem æquatio fluxionalis ſeu differentialis, aliquando duas ſolutiones, omninò ab invicèm diverſas, ſortiatur; quod primò detexit *Clairaut*, *Mem. Acad.* 1734, et poſteà confirmarunt *Euler*, *D'Alembert*, *De La Place*, *De La Grange*, et *Le Gendre*. Vide M. R. Append. Vol. 28, p. 537.

Hujuſmodi eſt æquatio differentialis $x\,dy\,dx - dy^2 = y\,dx^2 - dy\,dx$, ſive ſecundùm notationem fluxionalem, $x\dot{x}\dot{y} - \dot{y}^2 = y\dot{x}^2 - \dot{y}\dot{x}$. Huic enim

enim ſolvendæ æquè reſpondent vel æquatio fluentialis $4y = x^2 + 2x + 1$, vel $2ax - 2x = -4y + 1 - a^2$.

Similitèr quoque æquationem differentialem $a\,dy^2 + x\,dy^2 - y\,dy\,dx = x\,dy\,dx - y\,dx^2$ æquè ſolvent, vel $\frac{x}{\sqrt{y}} - \sqrt{y} = \sqrt{4a}$, vel $b^2y - 2bx + 2by = -4a$.

Supereſt ut brevitèr proponere aggrediar applicationem Fluxionum ad *Logarithmotechniam*, planiùs ut ſpero, et pleniùs, quam hactenùs; ob ingentem Logarithmorum utilitatem in Calculis Trigonometricis, &c. abbreviandis; ídque libentiùs, quòd inter celeberrimorum *Scriptorum Logarithmicorum* congeſta opera locum habere permittatur hæc *Analyſis Fluxionum*, mole quidem parva, ſed quæ tamèn, pro rerum difficultate, ſubtilitate, varietate, et pondere, (quæ fortaſſe fuſiùs explicari debuiſſent,) quantumvis impar, tanto honore non prorsùs indigna videatur.

De Fluxionibus Logarithmorum.

---

120. *Logarithmi*, quaſi λογων αϱιθμοι, "*Rationum numeri exponentes*" ſunt *numeri arithmeticè proportionales, numeris geometricè proportionalibus reſpondentes*, ſeu ſecundùm definitionem vulgatam, "*Numerorum proportionalium comites æquidifferentes*:" ut in ſeriebus inſequentibus Logarithmorum vulgarium:

Numeri	&c.	10^3,	10^2,	10^1,	10^0,	10^{-1},	10^{-2},	10^{-3},	&c.
	vel								
		1000,	100,	10,	1,	·1,	·01,	·001.	
Logarithmi		3,	2,	1,	0,	− 1,	− 2,	− 3.	

De utilitate inſigni Logarithmorum in omni genere Computorum Arithmeticorum, Trigonometricorum, abbreviando operationes *Multiplicationis, Diviſionis, Involutionis* et *Evolutionis*, hoc loco monuiſſe tantùm ſufficiat: adeat harum rerum ſtudioſus librum anteà memoratum, cui titulus *Scriptores Logarithmici*, à viro cl. *Maſeres* congeſtum et editum.

121. In omni ſyſtemate Logarithmorum, ſtatuitur logarithmus *unitatis* eſſe nihil, aut 0. In omni ſyſtemate igitùr, *ſumma logarithmorum cujuſvis numeri logiſtici, ejúſque reciproci*, cum ſint æquales et contrariis ſignis affecti, fiet 0, ſeu *evaneſcet*.

In

In Systemate Logistico vulgari, seu *Briggiano*, sumitur *unitas* pro logarithmo numeri 10; at in Systemate Hyperbolico, seu *Neperiano*, *unitas* prodit logarithmus numeri 2·7182818, &c.

122. In quovis autem systemate, numerus qui crescit vel decrescit eâdem ratione ac logarithmus ejus, fit quasi *modulus* istius systematis: erítque *modulus unius systematis ad modulum alterius, ut logarithmus dati numeri in uno ad logarithmum ejusdem numeri in altero.*

Sic in systemate vulgari, logarithmus numeri 10 est 1; at in hyperbolico, logarithmus 10 est 2·3025850, &c. In hyperbolico, item modulus est 1; undè prodit modulus systematis vulgaris $= \frac{1 \cdot 0000000}{2 \cdot 3025850} = 0 \cdot 4342944$.

Erítque *ratio modularis*, in systemate hyperbolico 1 ad 1, sive ratio æqualitatis; at in vulgari, 0,4342944 ad 1, nempè ratio data.

123. *Fluxiones logarithmorum*, fluxionibus *ordinis secundi* ascribendæ sunt; cum ipsi logarithmi *fluxionibus primis* respondent: ut ex definitionibus constat.

Methodus hásce inveniendi propositionibus insequentibus fundatur.

124. *Prop.* 1. *Quantitatum logisticarum fluxiones sunt ipsis quantitatibus proportionales.*

Nam quoniam quantitates ipsæ proportione *geometricâ* crescunt vel decrescunt, erunt differentiæ earum ipsis quantitatibus proportionales; per conversam *Lem.* 1. *Lib.* II. Princip. Et proinde, fluxiones, quæ variantur ut hæ differentiæ, erunt quoque ipsis quantitatibus proportionales. Q. E. D.

125. *Prop.* 2. *Fluxio quantitatis logisticæ est ad fluxionem logarithmi ejus, ut ipsa quantitas ad modulum systematis logistici.*

Per propositionem præcedentem est n, fluxio quantitatis N, ad m, fluxionem quantitatis M, ut N ad M. Designet M *modulum*, et quoniam m in hoc casu est constans, substitui potest pro fluxione logarithmi N quoque, cum hujus logarithmus est in datâ quâdam ratione ad modulum. Erit igitùr n, fluxio ipsius N ad l, fluxionem logarithmi ejus, ut N ad M. Q. E. D

126. *Prop.* 3. *In quovis systemate logarithmorum, fluxio logarithmi cujusvis dati æquatur fluxioni ipsius quantitatis logisticæ, in modulum ductæ, et per ipsam quantitatem divisæ.*

Nam (per *præced.*) est $n : l :: N : M$; et invertendo, $l : n :: M : N$, sive $l = \frac{nM}{N}$. Q. E. D.

127. *Prop.*

127. *Prop.* 4. *In ſyſtemate hyperbolico, fluxio logarithmi cujuſvis dati æquatur fluxioni ipſius quantitatis logiſticæ, per ipſam quantitatem diviſæ.*

Nam in hoc ſyſtemate, $M = 1$, et proinde, $\dot{l} = \frac{\dot{n}}{N}$. Q. E. D.

Idem quoque *geometricè* illuſtrare licet ex naturâ arearum *hyperbolæ rectangulæ*, inter curvam, aſymptotas et ordinatas intercluſarum. Deſignantibus enim x aſymptoti abſciſſam; y, ordinatam; et m, quadratum conſtans, contentum utique aſymptotis et parallelis à vertice ductis, ſeu modulum; (per naturam hyperbolæ,) erit $xy = m$; adeóque, $y = \frac{m}{x}$; et multiplicando utrinque per $\dot{x}$, erit $\dot{x}y = \frac{\dot{x}m}{x}$, ſive ob $m = 1$, $= \frac{\dot{x}}{x}$; ſed $\dot{x}y$ eſt fluxio logarithmi dati; (per naturam hyperbolæ et logarithmorum communem). Ergo, &c. Q. E. D.

Hinc manifeſtum eſt quòd *hyperbola rectangula* cujus axis eſt aſymptotos, eſt curva logiſtica, ſeu curva ad deſignandum logarithmos, et eos quaſi ante oculos ponendum, accommodata: creſcentibus enim abſciſſis in progreſſione *Arithmeticâ*, decreſcent ordinatæ in progreſſione *Geometricâ*.

De Fluxionibus Quantitatum Exponentialium.

128. *Quantitates exponentiales* ſunt aliarum quantitatum poteſtates quarum indices ſunt variabiles; ſicut vel e^x, vel y^x; quarum prima eſt poteſtas quantitatis conſtantis e variabili indice x deſignata, altera verò eſt poteſtas quantitatis variabilis y, variabili indice x paritèr deſignata.

Caſ. 1. Fiat $e^x = z$. Erítque $log.\ z = log.\ e, \times x$; adeóque per præced. $\frac{\dot{z}}{z} = \log.\ e, \times \dot{x}$; et proinde, $\dot{z} = \log.\ e \times \dot{x}z, = \log.\ e \times \dot{x}e^x$. Sive *fluxio quantitatis exponentialis* (e^x) *æquatur ipſi quantitati* (e^x), *in indicis fluxionem* ($\dot{x}$), *et lateris logarithmum* (log. e) *continuè ductæ.* Q. E. D.

Caſ. 2. Fiat $y^x = z$. Erítque log. $z = \log.\ y \times x$; adeóque $\frac{\dot{z}}{z} = \log.\ y \times \dot{x} = \frac{\dot{y}}{y} \times \dot{x}$; et proinde, $\dot{z} = \frac{\dot{y}}{y} \times \dot{x}z = \frac{\dot{y}}{y} \times \dot{x}y^x = \dot{y} \times \dot{x}y^{x-1}$. Sive *fluxio quantitatis exponentialis variabilis* (y^x) *æquatur fluxioni lateris* ($\dot{y}$) *in fluxionem indicis* ($\dot{x}$) *et co-efficienti* (y^{x-1}) *continuè ductæ.* Q. E. D.

N. B. In

N. B. In demonstratione hujus casûs, ex *Hutton's Mathematical Dictionary* excerptâ, occurrit *erratum* preli haud contemnendum, + pro =, bis. Art. *Fluxions*, p. 489.

De Logarithmis Imaginariis.

129. Logarithmi quantitatum impossibilium, ut $\sqrt{-a^2} = a\sqrt{-1}$, appellantur *Imaginarii*; quales sunt fluentes fluxionum quarundam impossibilium, sicut $\frac{\dot{x}}{x\sqrt{-1}}$. Nam quoniam fluxionis $\frac{\dot{x}}{x}$ fluens est logarithmus *verus* quantitatis realis x; pari ratione, erit fluxionis $\frac{\dot{x}}{x\sqrt{-1}}$ fluens, quoque logarithmus *imaginarius* quantitatis impossibilis $x\sqrt{-1}$.

Veruntamen ubi tales quantitates in problematum solutionibus occurrunt, facilè transmutari possunt in *arcus circulares*, aut *sectores veros*, ex *hyperbolicis impossibilibus*. Vide MACLAURIN, *Of the Analogy betwixt Circular Arcs and Logarithms*; *Fluxions*, § 762 :—PLAYFAIR, *On the Arithmetic of Impossible Quantities*; *Phil. Trans.* an. 1778, p. 318; or, *Analys. Æquat.* § 223, et *Scriptores Logarithmicos*, tomum secundum, in paginis 381, 382, &c - - - ad 585, ubi methodus quam invenit celeberrimus Leibnitzius anno 1676 pro extensione duarum regularum à Cardano olìm traditarum de radicibus æquationum cubicarum formæ $x^3 + qx = r$ in omnibus earum casibus, et formæ $x^3 - qx = r$ in primo earum casu, seu cum $\frac{rr}{4}$ est major $\frac{q^3}{27}$, ad secundum casum formæ secundæ, in quo $\frac{rr}{4}$ est minor ipsâ $\frac{q^3}{27}$ (qui *casus irreducibilis* à scriptoribus Algebræ appellatur,) fusè et accuratè explicatur.

130. Analogiam inter *Methodum Fluxionum* et *Logarithmorum*, ex recentioribus primus detexisse videtur sagacissimus *Kepler*; qui logarithmos ritè definivit "*mensuras rationum*," sive "*numeros ratiuncularum inter rationem numeri alicujus ad unitatem intercedentium*." Cæterùm antiquis penitùs latuisse hanc analogiam, haud facilè persuadear, vel ex solo opere celeberrimo *Apollonii*, *De Sectione Rationis*; quâcum ad amussim quadrat hæc expositio *Kepleriana*; atque, non injuriâ, *Apollonio* forsàn debuit editor ejus, sagacissimus *Halley*, methodum suam egregiam, sed paulò reconditiorem exquirendi et construendi logarithmos, sive "*numeros rationum exponentes*," (optimâ nominis definitione usus), per varias combinationes et *sectiones rationum*, series logisticas citissimè convergentes instituendo; qui omnium primus, ut fertur, *problema inversum* extudit: *dato logarithmo invenire numerum ejus logisticum*.

131. Methodum *Halleianam*, celeberrimam at perdifficilem quidèm, acutiùs exposuit *Jones*, in *Synopsi Palmariorum Matheseos*; plenissimè autem et luculentissimè pervestigavit analysta ex peritissimis *Maseres*, in magno suo repositorio *Scriptorum Logarithmicorum*, tomo primo in paginis 235, 236, &c - - - 383, et tomo secundo in paginis 76, 77, &c - - - - 152.

132. Accedit ultimò TABULA præcipuarum FORMULARUM FLUXIONALIUM et FLUENTIALIUM hactenùs repertarum, ex Cl. *Hutton's Mathematical Dictionary* excerpta.

Formulæ Fluxionum atque Fluentium.

	Fluxionum.	*Fluentium.*
I.	$x^{n-1}\dot{x}$ - - - - - -	$\frac{1}{n}x^{n}$.
II.	$(a \pm x^{n})^{m-1}x^{n-1}\dot{x}$ - - - - -	$\pm\frac{1}{mn}(a \pm x^{n})^{m}$.
III.	$\frac{x^{mn-1}\dot{x}}{(a\pm x^{n})^{m+1}}$ - - - - -	$\frac{1}{mna}\times\frac{x^{mn}}{(a\pm x^{n})^{m}}$.
IV.	$\frac{(a\pm x^{n})^{m-1}\dot{x}}{x^{mn+1}}$ - - - - -	$\frac{-1}{mna}\times\frac{(a\pm x^{n})^{m}}{x^{mn}}$.
V.	$mx^{m-1}y^{n}\dot{x}+ny^{n-1}x^{m}\dot{y}$. vel $(m\dot{x}y + nx\dot{y})x^{m-1}y^{n-1}$. vel $(\frac{m\dot{x}}{x}+\frac{n\dot{y}}{y})x^{m}y^{n}$.	- - $x^{m}y^{n}$.
VI.	$mx^{m-1}y^{n}z^{r}\dot{x}+nx^{m}y^{n-1}z^{r}\dot{y}+rx^{m}y^{n}z^{r-1}\dot{z}$ vel $(m\dot{x}yz + nx\dot{y}z + rxy\dot{z})x^{m-1}y^{n-1}z^{r-1}$ vel $(\frac{m\dot{x}}{x}+\frac{n\dot{y}}{y}+\frac{r\dot{z}}{z})x^{m}y^{n}z^{r}$.	$x^{m}y^{n}z^{r}$.

	Fluxionum.	Fluentium.
VII.	$\frac{\dot{x}}{x}$ - - - -	Log. x.
VIII.	$\frac{x^{n-1}\dot{x}}{a \pm x^{n}}$ - - -	$\pm \frac{1}{n}$ log. of $a \pm x^{n}$.
IX.	$\frac{x^{-1}\dot{x}}{a \pm x^{n}}$ - - - -	$\frac{1}{na}$ log. of $\frac{x^{n}}{a \pm x^{n}}$.
X.	$\frac{x^{\frac{1}{2}n-1}\dot{x}}{a - x^{n}}$ - - - -	$\frac{1}{n\sqrt{a}}$ log. of $\frac{\sqrt{a} + \sqrt{x^{n}}}{\sqrt{a} - \sqrt{x^{n}}}$.
XI.	$\frac{x^{\frac{1}{2}n-1}\dot{x}}{a - x^{n}}$ - - -	$\frac{2}{n\sqrt{a}} \times$ arc. tang. $\frac{\sqrt{x^{n}}}{a}$, vel $\frac{1}{n\sqrt{a}} \times$ arc. cosin. $\frac{a - x^{n}}{a + x^{n}}$.
XII.	$\frac{x^{\frac{1}{2}n-1}\dot{x}}{(\pm a + x^{n})^{\frac{1}{2}}}$ - - -	$\frac{2}{n}$ log. $\sqrt{x^{n}} + \sqrt{\pm a + x^{n}}$.
XIII.	$\frac{x^{\frac{1}{2}n-1}\dot{x}}{(a - x^{n})^{\frac{1}{2}}}$ - - -	$\frac{2}{n} \times$ arc. sin. $\frac{\sqrt{x^{n}}}{a}$, vel $\frac{1}{n} \times$ arc. vers. sin. $\frac{2x^{n}}{a}$.
XIV.	$\frac{x^{-1}\dot{x}}{(a \pm x^{n})^{\frac{1}{2}}}$ - - -	$\frac{1}{n\sqrt{a}}$ log. $\frac{\pm\sqrt{a \mp x^{n}} \pm \sqrt{a}}{\sqrt{a \pm x^{n}} + \sqrt{a}}$.
XV.	$\frac{x^{-1}\dot{x}}{(-a + x^{n})^{\frac{1}{2}}}$ - - -	$\frac{2}{n\sqrt{a}} \times$ arc. sec. $\sqrt{\frac{x^{n}}{a}}$, vel $\frac{1}{n\sqrt{a}} \times$ arc. cosin. $\frac{2a - x^{n}}{x^{n}}$.

Fluxio

	Fluxionum.	*Fluentium.*
XVI.	$(dx - x^2)^{\frac{1}{2}} \times \dot{x}$	$\frac{1}{2}$ circ. ſegm. ad diam. d et verſ. ſin. x.
XVII.	$c^{nx}\dot{x}$	$\frac{c^{nx}}{n \log. c}$.
XVIII.	$\dot{x}y^x \log. y + xy^{x-1}\dot{y}$	y^x.
XIX.	$(a + x^n)^m x^{rn-1}\dot{x}$	$\frac{(a+x^n)^{m+1} x^{rn-n}}{ns} \times \left(\frac{1}{1} - \frac{r-1 \cdot a}{s-1 \cdot x^n} + \frac{r-1 \cdot r-2 \cdot a^2}{s-1 \cdot s-2 \cdot x^{2n}}, \text{\&c.}\right)$

De Formularum Applicatione.

---

133. Methodus applicationis hujuſmodi eſt.

Conferatur fluxio propoſita cum formulis; nempè, partes correſpondentes cum correſpondentibus fluxionis tabularis quâcum conſentit; et hiſce valoribus ſubſtitutis in formulam fluentis generalem, invenietur fluens quæſita.

Ex. gr. 1. *Invenire fluentem fluxionis* $3x^{\frac{5}{3}}\dot{x}$. Conſentit hæc cum *primâ* formulâ. Adeóque, propter $n - 1 = \frac{5}{3}$, erit $n = \frac{5}{3} + 1 = \frac{8}{3}$; erítque $\frac{1}{n}x^n = \frac{3}{8}x^{\frac{8}{3}}$. Et proinde fluens quæſita, multiplicando per 3, co-efficientem fluxionis datæ, erit $3 \times \frac{3}{8}x^{\frac{8}{3}} = \frac{9}{8}x^{\frac{8}{3}}$.

2. *Invenire fluentem fluxionis* $\frac{x^2\dot{x}}{1+x^3}$. Conſentit hæc cum *octavâ* formulâ. Nam prodit $\frac{x^{n-1}\dot{x}}{a+x^n} = \frac{x^2\dot{x}}{1+x^3}$, itémque $n = 3$, et $a = 1$; quibus ſubſtitutis in formulam fluentis generalem, prodit fluens quæſita $\frac{1}{3}\log. \overline{1+x^3}$.

 3. *In-*

3. *Invenire fluentem quantitatis* $5\dot{x}x^2\sqrt{c^3 - x^3}$ seu $5\dot{x}x^2(c^3 - x^3)^{\frac{1}{2}}$. Quadrat hæc cum formulâ *secundâ*. Erit igitur $a = x^3$; $m - 1 = \frac{1}{2}$, seu $m = \frac{3}{2}$, et $n = 3$; quibus substitutis, prodit fluens quæsita, $5 \times \frac{1}{\frac{3}{2} \times 3}(c^3 - x^3)^{\frac{3}{2}} = \frac{10}{9}(c^3 - x^3)^{\frac{3}{2}}$.

134. Ubi quantitas fluxionalis proposita nullis ex his formulis finitis respondeat, inveniri potest ejus fluens quàm proximè, *resolvendo propositam in seriem infinitam; vel per divisionem, vel per theorema binomium; et inveniendo singulorum terminorum fluentes; quorum summa ad libitum continuatorum, dabit fluentem quæsitam, proximè aut quam proximè.*

4. Invenire fluentem $\frac{1 - x}{1 + x - x^2}\dot{x}$. Dividendo numeratorem per denominatorem, prodit $\dot{x} - 2x\dot{x} + 3x^2\dot{x} - 5x^3\dot{x} + 8x^4\dot{x}$, &c. Et inventis singulorum fluentibus, prodit summa $x - x^2 + x^3 - \frac{5}{3}x^4 + \frac{8}{5}x^5$, &c. fluens quæsita.

De correctione fluentium per regulas aut per formulas inventarum, vide suprà, § 118.

APPEN-

APPENDIX I.

DE ANALYSI ANTIQUÂ.

Antiquam exquirite Matrem. VIRG.

APPENDIX I.

DE ANALYSI ANTIQUÂ.

§ 135. ANALYSIN Antiquam, qualem exercebant clariſſimi Geometræ veteres, *Euclides*, *Archimedes*, *Apollonius*, &c. jamdudùm laudavimus; optimam ſanè, judice in hiſce verſatiſſimo, *Halley*, ANALYS. ÆQUAT. § 128. Et quàm curiosâ felicitate eandem excoluit et auxit *Newtonus*, ex operibus ejus philoſophicis abundè conſtat. Placet igitùr, hoc tempore, ubi labefactantur PHILOSOPHIÆ NATURALIS vel ipſa PRINCIPIA MATHEMATICA, et juventuti noſtræ horum ſtudioſæ potiùs commendantur, (utpotè majoris evidentiæ, et rigoris geometrici,) "*functiones quædam analyticæ*," "*fluxiones hypotheticæ*," è *Scholâ Gallicâ* novatricè profluentes, eximiam illam *Præfationem Pappi Alexandrini*, ex Editione Halleianâ *Apollonii de Sectione Rationis*, hoc loco edere ab integro.

136. " Locus *de Reſolutione* inſcriptus, *Hermodore* fili, ut paucis dicam, propria quædam eſt materia in eorum uſum deſignata qui, perceptis *communibus elementis*, in GEOMETRIA facultatem ſibi deſiderant *inveſtigandi ſolutiones problematum*, et in hunc finem ſolummodò utilis. Traditur autem à tribus viris, *Euclide*, nempe, Elementorum ſcriptore, *Apollonio Pergæo*, et *Ariſtæo* ſeniore.

1. " Procedit verò, per *Methodum Reſolutionis et Compoſitionis*. *Reſolutio* (αναλυσις) autem eſt *methodus quæ à quæſito quaſi jam conceſſo, per ea quæ deinde conſequuntur ad concluſionem aliquam cujus ope compoſitio fit*, (επι τι ὁμολογημενον εν συνθεσει) *perducamur*. In reſolutione enim quòd quæritur *ut jam factum* ſupponentes, ex quo antecedente hoc conſequatur, expendimus; iterúmque quodnam fuerit hujus antecedens; atque ita deinceps; uſque, dum in hunc modum regredientes, in *aliquid jam cognitum*, locóque *principii* habitum, incidamus. Atque hic proceſſus *Analyſis* vocatur, quaſi dicas (αναπαλιν λυσιν) *Inverſa Solutio*. Quid ſit *Analyſis*, ſive *Reſolutio*.

" E con-

Quid sit *Synthesis*, sive *Compositio*.

"E contrario autem, in *compositione* (εν συνθεσει,) cognitum illud in *resolutione* ultimo loco acquisitum, ut *jam factum* præmittentes, et quæ ibi consequentia erant, hìc ut antecedentia, naturali ordine disponentes, atque inter se conferentes, tandem ad *constructionem* quæsiti pervenimus. Hoc autem vocamus *Synthesin*.

"Duplex autem est *analyseos* genus: vel enim est veri indagatrix, dicitúrque *Theoretica*, vel propositi investigatrix, ac *Problematica* vocatur. In *theoretico* autem genere, *quod quæritur, reverà ita se habere supponentes*; *ac deinde per ea quæ consequuntur, quasi vera sint*, (ut sunt *ex hypothesi*,) *argumentantes, ad evidentem aliquam conclusionem* (επι τι ὁμολογȣμενον) *procedimus*. Jam si *conclusio* illa *vera* sit, *vera* est quoque *propositio* de quâ quæritur; ac demonstratio reciprocè respondet *analysi*: si verò in *falsam* conclusionem incidamus, *falsum* quoque erit de quo quæritur.

"In *problematico* verò genere, *quod proponitur, ut jam cognitum sistentes, per ea quæ exinde consequuntur tanquam vera, perducimur ad conclusionem aliquam*. Quòd si conclusio illa *possibilis* sit ac πορισικη (quod mathematici *datum* appellant) *possibile* quoque erit quod proponitur: Et hìc quoque demonstratio reciprocè respondet *analysi*: si verò incidamus in conclusionem *impossibilem*, erit etiàm problema *impossibile*.

"*Diorismus* autem (διορισμος), sive *determinatio*, est quâ discernitur *quibus* conditionibus, et quot modis, problema effici potest. Atque hæc de *resolutione* et *compositione* dicta sunto."——

137.—"Verùm perpendendum est (notat *Halley*,) aliud esse problema *aliqualitèr* resolutum dare, quod modis variis plerúmque fieri potest; aliud, *methodo elegantissimâ* id efficere: *Analysi* brevissimâ et simùl perspicuâ; *Synthesi* concinnâ et minimè operosâ. Hoc veteres præstitisse, argumento est *Apollonii* Liber *De Sectione Rationis*."

138.—"Ex his, credo, manifestum est (monet quoque *Newtonus*, *De Resolutione Quæstionum Geometricarum*,) quid sibi velint *Geometræ*, cum jubent, '*putes factum esse quod quæris*:' Nullo enim inter cognitas et incognitas quantitates habito discrimine, *quaslibet* ad ineundum calculum *assumere* potes, quasi *omnes* ex præviâ solutione essent notæ; et non ampliùs de *solutione problematis* sed de *probatione solutionis* ageretur."—"Convenit ut fingas quæstionem de *ejusmodi datis et quæsitis* propositam esse per quas arbitreris te posse ad æquationem *facillimè* pervenire."—"Cum varios ordines quibus termini quæstionis evolvi possint perspexeris, *E syntheticis quoslibet adhibe* assumendo lineas tanquam *datas*, à quibus ad *alias facillimus videtur progressus*, et *ad ipsas vicissim difficillimus*."—"Manuducit *analysis* ad *compositionem*: sed compositio non priùs verè fit, quam liberatur ab omni analysi. Insit compositioni vel minimum analyseos, et compositionem veram nondùm assecutus es. *Compositio* in se perfecta est, et à mixturâ *speculationum analyticarum* abhorret."

Hæc

Hæc *Newtonus*, in "LIBRO" illo verè aureo, "DE COMPOSITIONE ET RESOLUTIONE," quem concinnavit, ut firmior "*ſtaret ſuper vias antiquas*" Juventus *Britannica*; et quo meliùs obviàm iret pravæ iſti novitati ex Scholâ *Carteſianâ* graſſanti, quâ *Carteſius*, "mixturis ſpeculationum *analyticarum*" nimìs indulgendo, evidentiam et certitudinem *Geometriæ*, primus, obſcurare et labefactare cœpit; et proinde, quaſi per antitheſin operi *Carteſiano*, cui nomen erat, per abuſum, "GEOMETRIA:" Librum ſuum, *Newtonus*, titulo magìs idoneo, ARITHMETICÆ UNIVERSALIS inſcripſit.—Atque ad hunc librum, mole parvum, at pondere et ſubtilitate materiæ eximium ("cui forsàn ſecundum vix inveniatur, niſi opus analyticum *Algebra Clairautiana*") interpretandum atque illuſtrandum, jamdudùm edebam ANALYS. ÆQUATIONUM, anno 1784. Vide *Præfat.* illius operis.

139. Methodum *Analyticam* et *Syntheticam* non ſolùm in *Matheſi*, ſed etiàm in *Mechanicâ Rationali*, optimè exponit *Newtonus*, ſub finem *Opticorum*.

"As in *Mathematics*, ſo in *Natural Philoſophy*, the inveſtigation of *difficult* things by the Method of *Analyſis*, ought ever to precede the Method of *Compoſition*.

"*This Analyſis* conſiſts in making *experiments* and *obſervations*, and in drawing *general concluſions* from them by *Induction*; and admitting of no *objections* againſt the concluſions, but ſuch as are taken from *experiments* or other *certain truths*: for *hypotheſes* are not to be regarded in EXPERIMENTAL PHILOSOPHY. And although the arguing from experiments and obſervations be *no demonſtration* of general concluſions, yet it is the *beſt way* of arguing which the nature of things admits of, and may be looked-upon as ſo much the ſtronger, by how much the induction is more general: and if *no exception* occur from phenomena, the concluſion may be pronounced generally. But if, at any time afterwards, any exception ſhall occur from *experiments*, it may then begin to be pronounced with *ſuch exceptions as occur.*"

"By this way of *Analyſis*, we may proceed from *Compounds* to *Ingredients*, and from *Motions* to the *Forces* producing them; and, in general, from *Effects* to their *Cauſes*; and from *particular* cauſes to *more general* ones, till the argument end in the moſt general:—this is the Method of *Analyſis*."

"And the *Syntheſis* conſiſts in *aſſuming* the *Cauſes* diſcovered and eſtabliſhed, as *Principles*; and by them *explaining* the *phænomena* proceeding from them, and *proving* the explanations."

And in the Preface to his PRINCIPIA, he gives the following elegant ſummary of this ſtatement:—

 "*Omnis*

" *Omnis* enim PHILOSOPHIÆ *difficultas* in eo verſari videtur, ut à *phænomenis motuum* inveſtigemus *Vires Naturæ*; deinde, ab *his viribus*, demonſtremus *phænomena reliqua*."

140. And he furniſhes the following inſtances of his application of this method, in thoſe two immortal works, his OPTICS and PRINCIPIA:

" In the two firſt books of theſe OPTICS, I proceeded, by *this Analyſis*, to diſcover and prove the *original differences* of the RAYS OF LIGHT, in reſpect of *Refrangibility*, *Reflexibility*, and *Colour*, &c.—And theſe diſcoveries being *proved*, may be aſſumed, in the Method of *Compoſition*, for *explaining the phænomena* ariſing from them: an inſtance of which I gave in the end of the firſt book, [in the ſolution of the colours and figure of the *Rainbow*, from the refraction and reflexion of the Sun's rays in drops of falling rain.]

" In libro autem tertio [PRINCIPIORUM] exemplum hujus rei propoſuimus per explicationem SYSTEMATIS MUNDANI: Ibi enim, ex *phænomenis cœleſtibus*, per propoſitiones in libris prioribus *mathematicè* demonſtratas, derivantur *vires gravitatis*, quibus corpora ad Solem et Planetas ſingulos tendunt: deinde, ex *his viribus*, per propoſitiones etiàm *Mathematicas*, deducuntur motus *Planetarum*, *Cometarum*, *Lunæ* et *Maris*."

141. And he expreſsly ſtates that he adopted this method from the Ancients:—

" Cum VETERES *Mechanicam* [*Rationalem*, quæ per demonſtrationes *accuratè* procedit,] (uti auctor eſt *Pappus*,) in *rerum naturalium* inveſtigatione maximi fecerint; et RECENTIORES, miſſis *formis ſubſtantialibus*, et *qualitatibus occultis*, *phænomena* NATURÆ ad *leges mathematicas* revocare aggreſſi ſint: *viſum eſt in hoc tractatu* MATHESIN *excolere, quatenùs ea ad* PHILOSOPHIAM *ſpectat*."

How highly NEWTON admired the *Ancient Geometricians*, we learn from Dr. PEMBERTON:—

" He often cenſured the handling of *Geometrical* ſubjects by *Algebräick* calculations; and his Book of Algebra he called by the name of *Arithmetica Univerſalis*, in oppoſition to the injudicious title of *Geometry*, which DES CARTES had given to the Treatiſe in which he ſhews how the Geometrician may aſſiſt his invention by ſuch kind of computations. He frequently praiſed SLUSIUS, BARROW, and HUYGENS for not being influenced by the *falſe taſte* which then began to prevail: he uſed to recommend the laudable attempt of HUGO D'OMERIQUE to reſtore the *Ancient Analyſis*; and very much eſteemed APOLLONIUS's book *De Sectione Rationis*, for giving us a clearer notion of that Analyſis than we had before. Dr. BARROW may be eſteemed as having ſhewn a compaſs of *invention* equal, if not ſuperiour, to any of the Moderns, our Author only excepted: but

NEWTON

NEWTON particularly recommended HUYGENS's ſtyle and manner; he thought him *the moſt elegant of any mathematical writer of modern times, and the trueſt imitator of the Ancients.* Of *their* taſte and mode of demonſtration our Author always profeſſed himſelf a great admirer; and even cenſured himſelf for not following them yet more cloſely than he did: and ſpoke with regret of his miſtake, at the beginning of his Mathematical Studies, in applying himſelf to DES CARTES and other *Algebräick* writers, before he had conſidered *the Elements* of EUCLID with that attention which ſo excellent a writer deſerves."

Abſolutis hactenùs VINDICIIS NEWTONIANIS, quatenùs *Elementa Metaphyſica* et *Principia Mathematica* reſpiciunt, (quibus fundantur opera ejus eximia Analytica et Philoſophica) adversùs objectiones *Gallorum*, *Germanorum*, *Anglorúmque*, horum evidentiam et certitudinem, aut malignè, aut imperitè, aut oſcitantèr, inſectantium; ſupereſt *altera* et *acrior cura*, ut repellatur crimen multo gravius, "*facinúſque majoris abollæ*," quaſi PHILOSOPHIÆ atque THEOLOGIÆ cultor ille, omnium modeſtiſſimus et ſaniſſimus, fuiſſet et ipſe fautor dogmatis praviſſimi atque turpiſſimi quod *Materialiſmum* vocant, Naturam *Mentis Humanæ* merè mechanicam ſtatuentis, et "*divinæ particulam auræ*," (etiàm ab antiquiſſimis agnitæ,) extinguere geſtientis; ipſum NUMEN SUMMUM—OPTIMUM, SAPIENTISSIMUM, MAXIMUM et POTENTISSIMUM omnium, quaſi *Animam Mundi*, *Æthercam*, pro nefas! abſurdè et impiè fingentis, et deliramenta "*inſanientis ſapientiæ*," iterùm recoquentis, per Philoſophiam pravam et Mythologiam *Leibnitzianam*, graviſſimóſque errores inde prognatos; hoc *ſæculo rationis* (ut dicitur) infauſto, *Revelationem* aſpernante.

APPENDIX II.

DE ÆTHERE VIBRATORIO,—DE MODO SENTIENDI,

ET

DE ENTE SUPREMO.

"AUCTORIBUS UTI OPTIMIS *in omnibus causis, et debet et solet valere plurimum: et primum quidem,* OMNI ANTIQUITATE; *quæ, quo propriùs aberat ab ortu et divinâ progenie, hoc meliùs ea, fortasse, quæ erant vera, cernebat.*" CIC.

Ουκ εγνω ὁ Κοσμος, δια της σοφιας, ΤΟΝ ΘΕΟΝ.

"*Non novit Mundus, Sapientiæ [humanæ] ope,* DEUM." PAUL.

APPENDIX II.

DE ÆTHERE VIBRATORIO.

"*CE grand homme* (NEWTON) *voyoit à travers d'un voile, ce qu'un autre ne distingue qu'à peine avec un Microscope!*" DAN. BERNOULLI *.

142. Nuper, cum perlegerim opus eximium, perutile, tempestivum, à viris bonis ubique laudatum,—"*Proofs of a Conspiracy against all the Religions and Governments of Europe*, &c. by *John Robison*, A. M. *Professor of Natural Philosophy*, &c. *Edinburgh*,"—incidi in censuram haud levem *Patris Philosophiæ Britannicæ*, p. 483.

"Were it possible for the departed soul of *Newton* to feel pain, he would surely recollect with regret that *unhappy hour*, when, provoked by Dr. *Hooke*'s charge of plagiarism, he first threw out his *whim* of a *Vibrating Æther*; to shew what might be made of an *Hypothesis:* (for Sir *Isaac Newton* must be allowed to have *paved the way* for much of the *Atomical Philosophy* of the Moderns.) Newton's *Æther* is assumed as a *fac totum* by every *precipitate sciolist*, who, in despite of logic, and in contradiction to all principles of mechanics, gives us *theories* of *muscular motion*, of *animal sensation*, and even of *intelligence* and *volition*, by the *undulations of ætherial fluids*.

"Not one of a hundred of these *theorists* can go through the fundamental theorem of all this doctrine, (the 47th *Prop.* of the *Second Book of the* PRINCIPIA); and not one of a thousand know that *Newton*'s investigation is *inconclusive:*—yet they talk of the effects and modifications of those undulations as familiarly as if they could demonstrate the propositions in *Euclid*'s *Elements*."

Talis viri ictus, PHYSICAM excolentis, et NEWTONUM quasi præceptorem adamare profitentis, acerbiùs lædunt: Antistiti ipsi scientiarum venerando,

* Vide Note (*e*) posthàc.

annon exclamare fas ſit, jamdudùm hujuſmodi crimina auſcultanti :—" Si id *Cinerem* aut *Manes* credis curare ſepultos (*a*)."

—" *Pol, me occidiſtis* AMICI !—
Ohe jam ſatis eſt !—Et TU, BRUTE !!!—

Telum imbelle jacis; imò, tuum in cerebrum facilè retorquendum, vibranti medium tranſverberans ictu."

143. Pluſquàm viginti retrò annis, Diſquiſitionem edebam, anno 1778, *Dublin, De Sonis* et *De Modificationibus Atmoſphæræ*, ſecundùm Theoriam *Newtonianam*,—primitias Studiorum meorum Philoſophicorum,—dum, apud Academicos *Dublinienſes*, *Socius* degerem. Et, cum iſta diſquiſitio forsàn nondum ad manus pervenerit Profeſſoris *Robiſon*, me operam vel illi viro clariſſimo déque Republicâ optimè merenti, haùd ingratam impenſurum ſpero, ſi exinde pauca excerpſerim, hanc rem ſpectantia.

Primo igitùr in limine, monendus eſt, *Newtonum*, haudquaquam " *opinionum* " ſeu *hypotheſeon* vanarum ſtudioſum, nec novum neque inauditum " *commentum* " de " *æthere* quodam *vibratorio* " finxiſſe ; ſed vetuſtum omninò, atque à philoſophis ſanioribus receptum : § 42.

Zeno, audiendi facultatem ità exponit, *Laert. Lib.* 7. § 158.—Ακουειν δε, του μεταξυ του τε φωνουντος και του ακουοντος, αερος πληττομενου σφαιροειδως, ειτα κυματουμενου, και ταις ακοαις προσπιπτοντος· ὡς κυματουται το ἐν τῃ δεξαμενῃ ὕδωρ, κατα κυκλους, ὑπο του εμβληθεντος λιθου.

" *Auditus* fit, ubi aer inter loquentem audientémque interpoſitus, pulſatur *ſphæricè*; deinde undulatur, auribúſque impingit : ſicuti undulatur *aqua* in ciſternâ, *ſecundùm circulos*, injecto lapide."

(*a*) Quantum à diſputationibus controverſiiſque abhorruit mite *Newtoni* ingenium, conſtabit ex Epiſtolâ ejus *ad Oldenburgh*, Oct. 24, 1676.

—" Five years ago, (1671,) when, at the perſuaſion of my friends, I had reſolved to publiſh a Treatiſe of the *Refraction of Light and Colours*, which I had then by me ; I began again to turn my thoughts to theſe *Serieſes* [for determining the Areas of Curves] ; and wrote likewiſe a Treatiſe upon them, that I might publiſh both together.—But having wrote you a letter upon the *Reflecting Teleſcope*, in which I explained briefly my notions on the *Nature of Light*, ſomething unexpected fell out, which made me imagine it concerned me to write to you ſpeedily about printing that Letter. Immediately upon the back of this, I met with *frequent interruptions* from ſeveral perſons' letters, taken up with *objections*, and *other* things, which *frighted* me entirely from executing my deſign ; and made me find fault with myſelf for my *imprudence*—that, by catching at *a ſhadow*, I had been ſo far *deprived of the peace and quiet of my own mind* ; *which is a thing of more ſubſtantial worth*."

At laſt *Newton* found out an expedient that rid him of much of this troubleſome and teazing correſpondence : when objections were ſtated againſt the accuracy or validity of his *Optical* experiments, he ſimply replied—" I *doubt the fact* ;"—" *repeat the experiment*,"—inſtead of entering into any diſcuſſion or vindication about his own.

Diogenis

Diogenis atque *Platonis* placita quoque, ſic refert *Plutarchus*, *De Placit. Philoſ.* Lib. 4. cap. 16.—*Διογενης, τȣ εν τῃ κεφαλῃ αëρος ὑπο της φωνης τυπ]ομενȣ και κινȣμενȣ· Πλατων, και ὁι απ᾽ αυτȣ, πληττεσθαι τον εν τῃ κεφαλῃ αëρα, τȣτον δε ανακλασθαι εις τα ἡγεμονικα, και γινεσθαι της ακοης την αισθησιν.*

" *Diogeni* [placet hoc effici] aëre intrà caput à voce pulſato atque commoto. *Platoni*, ejúſque ſequacibus, aërem intrà caput pulſari, deinde ad partes *imperatorias* reflecti; atque hoc pacto ſenſum auditûs fieri."

Lucretius quoque phænomena *Lucis* et *Sonorum* diſertè confert: *Lib.* 4.

" Quod ſupereſt, non eſt mirandum quâ ratione,
Quæ loca per, nequeunt oculi res cernere apertas;
Hæc loca per, *voces* veniant *aureîſque laceſſant.*
Conloquimur clauſis foribus, quod ſæpe videmus:
Nimirùm, quia *vox per flexa foramina* rerum
Incolumis tranſire poteſt; *ſimulacra* renutant;
Perſcinduntur enìm, niſi recta foramina tranant,
Qualia ſunt vitri, ſpecies quâ travolat omnis.
   Prætereà, *partes in cunctas dividitur vox*;
Ex aliis aliæ quoniam gignuntur, ubi una
Diſſiluit ſemèl in multas exorta: (quaſi ignis
Sæpe ſolet ſcintilla ſuos ſe ſpargere in ignes);
Ergò *replentur loca vocibus*, *abdita retrò*,
Omnia quæ circùm fuerint, ſonitúque cientur:
At *ſimulacra* viis directis omnia tendunt,
Ut ſunt miſſa ſemèl; quaproptèr *cernere* nemo
Se ſuprà potis eſt, at *voces* accipere extrà."

Atqui hìc habes quaſi ſeminarium totius *Doctrinæ Sonorum*, rationalis et experimentalis, in difficillimâ parte difficillimi operis, nempè, in *Sectione octavâ*, *Lib.* 2. *Princip.* " *De Motu per fluida propagato* expoſitæ et demonſtratæ."

144. Haudquaquàm patitur hujus inſtituti ratio, de Doctrinâ ſubtiliſſimâ Pulſuum et Vibrationum Aëris, ſeu *Medii Ætherei*, jam diſſerere, quæ vel mathematicè doctos non parùm morantur; accuratiùs autem exponitur fuſiúſque illuſtratur tota hæc doctrina, in Diſquiſitione prædictâ *De Sonis.*

Unam ſaltèm obſervationem inde depromere non pigebit, p. 139, *de veritate propoſitionis illius fundamentalis* 47, à Profeſſore *Robiſon* denegatæ:

" *Le Sœur* et *Jacquier* veritatem propoſitionis deducunt ex vibrationibus partium *chordæ elaſticæ*, quæ peraguntur viribus in ratione diſtantiarum à medio vibrationum loco: ſed ſingulæ aëris particulæ motum chordæ *mutuantur* ſucceſſivè: ergò, inſtar chordæ ipſius, *moventur* viribus, &c.—At hæret deductio

nisi ostendere possent, *omnia corpora* sonitum edentia *legem* chordæ elasticæ in tremoribus suis servare, [*aërémque* similitèr agitari].

"*Rowning*, *Nat. Phil. Part* 2. p. 48, veritatem propositionis immediatè deducit ex *lege repulsivâ partium aëris* :—Nam particula quævis, pressione partium à tergo, versùs particulas antecedentes propelletur viribus quæ sunt ut intervalla centrorum reciprocè: atque ideò, ut distantiæ hujus particulæ à primo suo loco directè."

Ex hâc lege *vis elasticæ* seu *repulsivæ*, (quæ à *Newtono* mathematicè demonstratur, *Prop.* 23. *Lib.* 2. *Princip.* et experimentis confirmatur, § 4, *De Sonis*), elicitur conclusio, Quod *singulæ fluidi elastici tremuli particulæ, motu reciproco brevissimo euntes et redeuntes, accelerantur semper et retardantur in ratione distantiarum à medio vibrationum loco*: hoc est, *pro lege penduli in cyclo-eïde oscillantis*.

145. Demonstratio *Newtoniana*, quâ instituitur analogia hæc elegantissima inter *vibrationes aëris* et *oscillationes penduli*, longè subtilissima, est prorsus analytica, ex ipsis fontibus ANALYSIS ANTIQUÆ hausta:

"*Fingamus medium* [*elasticum*] *tali motu* [*vibratorio*] *à causâ quâcunque cieri; et videamus quid indè consequatur*, &c."

Propositionis autèm 47 ipsa conclusio, "*Aërem, pulsibus per medium elasticum propagatis, cieri seu agitari pro lege oscillantis penduli*," abundè confirmatur experimentis de variis sonorum generibus, tormentorum displosionibus, &c. factis ad investigandam *velocitatem sonorum*, (seu rationem secundùm quam rectà progrediuntur), in ITALIA, observantibus Academicis *Florentinis*; in ANGLIA, observantibus *Newtono*, *Derhamo*, &c.; in AMERICA AUSTRALI, observantibus Academicis *Gallis* atque *Hispanis*; et nupèr etiàm in AFRICA, propè ostium *Nili*, victoriâ illâ insigni navali apud Sinum *Aboukir*, Aug. 1, 1798, à classe Anglicâ sub imperio ducis magnanimi Horatii Nelson, de classe Reipublicæ Gallicæ reportatâ; ubi fragor navis *L'Orient*, explosæ, *Roset'am* pertingebat, justo post fulgorem visum intervallo temporis, ratione habitâ distantiæ et fluxûs uniformis sonorum (*b*).

(*b*) Si observationibus M. *Poussielgue*, *Gallorúmque* adstantium, tempore illo trepidationis bellicæ iniquo et infausto, fides est adhibenda, fragor navis *L'Orient*, in sinu *Aboukeriano* explosione combustæ, stationem suam apud *Rosettam* pertingebat tempore *duorum minutorum primorum* post fulgorem visum. At progagatur sonus mediâ velocitate 1142 pedum circitèr, tempore unius minuti secundi, et proindè 13 milliarium Anglicorum tempore unius minuti primi. Distabant igitùr *Galli* 26 milliaria Anglica à loco explosionis, *ad minimum*, recto cursu.

Augenda est enim hæc distantia, ob *calores æstivos* istius regionis, in ratione 1142, seu potiùs pedum Gallicorum 1070 ad 1101; sicùt pedum Gallicorum 1101, singulis minutis secundis, determinabatur velocitas sonorum ad insulam *Cayenne*; (intervallo 20230 hixapedarum *(toises)*, seu milliarium Anglicorum 25 ferè, geometricè mensurato à *Condamine* Academicísque Gallis; atque adeò accuratè ut, ex quinque experimentis factis per tormentorum explosiones, quatuor vix uno semi-secundo temporis ab invicèm discrepârunt. Distantia igitùr, hâc ratione aucta, correctior emerget milliarium 27 ferè. Atque iterùm forsàn augenda est, ob ingentem fragorem navis explosæ, intensissimos tremores, eosdémque forsàn principio latissimos, excitantis. Academicorum *Anglorum* certè æquum foret rem determinare, geometricè mensurando distantiam stationis apud *Rosettam*, à nave *L'Orient*; ad perficiendam aut corrigendam Theoriam *Newtonianam*, experimento adeò illustri, minimè negligendo.

Conſenſus igitùr theoriæ cum experimentis fidelitèr inſtitutis ubicunque regionum ſatìs confirmat *Analyſin Newtonianam* ritè inſtitutam fuiſſe; nam *veritas concluſionis infert veritatem præmiſſorum.*—Vide *De Sonis*, § 47; ubi datur Tabula Obſervationum *Florentinorum*, &c.; necnon *Append.* p. 71; ubi demonſtrationes harum propoſitionum valdè difficillium, 47 et 49, pleniùs exhibentur, atque notis illuſtrantur.

146. Nec eſt cur de *Æthere Vibratorio* cavillationes excitentur:—vox *æther* à Newtono uſurpatur pro *aëre puriſſimo*, *vaporibus* ſordibúſque heterogeneis defæcato:—"hi enim, (cum ſint alterius elateris et alterius toni,) vix, et ne vix quidèm, participant motum *aëris veri* quo ſoni propagantur." *Princip.* p. 373.

Hunc *ætherem*, ſeu *medium æthereum*, ingenti elatere (ſeu elaſticitate) eſſe prædìtum, docet *Newtonus*, *Optics*, p. 325, his verbis:

"That the *elaſtic force* of this medium is exceeding great, may be gathered from the ſwiftneſs of it's vibrations; *ſounds* move about 1140 Engliſh feet in a ſecond of time, and in 7 or 8 minutes of time they move about 100 Engliſh miles: *Light* moves from the Sun to us in about 7 or 8 minutes of time; which diſtance (ſuppoſing the Sun's horizontal parallax to be about 12″,) is about 70 millions of Engliſh miles; and by conſequence, above 700,000 times ſwifter than ſounds: and therefore the elaſtic force of this medium muſt be above 490,000 million of times greater than the elaſtic force of the air [at the earth's ſurface]"—in proportion to their reſpective rarities (*c*).

147. The aſtoniſhing *rarefaction* of which the air is capable is remarked, *Optics*, p. 341, from experiment and from computation:

"Mr. *Boyle* has ſhewn that air may be rarefied about 10,000 times in veſſels of glaſs; and the heavens are much emptier of air than any *vacuum* we can make below. For, ſince the air is compreſſed by the weight of the incumbent atmoſphere, and the denſity of the air is proportional (or it's rarity reciprocally proportional) to the force compreſſing it, it follows, by computation, [*Princip.* *Lib.* 2. *Prop.* 22. *Cor.* et *De Sonis*, § 14.] that, at the height of about 7½ Engliſh miles from the earth, the air is 4 times rarer than at the ſurface of the earth; and at the heights of 22½, 30, or 38 miles, it is reſpectively 64, 256, or 1024 times rarer, or thereabouts; and at the height of 76, 152, 228 miles, it

(*c*) *Newton*, *Principia*, p. 495, eſtimates the Sun's horizontal parallax, lower, at 10½."—And the calculations made ſince the Tranſit of Venus in 1761, reduce it further to 8″,6; which will enlarge the Sun's diſtance from us to upwards of 95 millions of miles; and conſequently raiſe the proportion of the elaſtic force of the ætherial medium to upwards of 900,000 million times greater than that of our air.

The calculation depends on the velocities of the pulſes (or propagation of the vibrations) of elaſtic fluids being in a compound ſubduplicate ratio of their elaſticities and rarities. *Princip.* II. 49. *Cor.* 2.

is about a million, a billion, or a trillion, times rarer; and ſo on:" inſomuch that, in the *Principia*, p. 512, *Newton* computes, that "a globe of our air one inch in diameter, at the elevation of one ſemidiameter of the earth (about 4000 miles), would, by it's expanſive power, fill all the planetary regions, far beyond the orbit of *Saturn*."

148. Non temère igitùr conjicit *Newtonus*, *Princip.* p. 530—"De *ſpiritu quodam ſubtiliſſimo corpora craſſa pervadente, et in iiſdem latente*; cujus vi et actionibus particulæ corporum ad minimas diſtantias ſe mutuò *attrahant*; et contiguæ factæ, *cohærent*; et corpora *electrica* agunt ad diſtantias majores, tàm repellendo quàm attrahendo corpuſcula vicina; et *lux* emittitur, reflectitur, refringitur, inflectitur et corpora calefacit; et *ſenſatio omnis* excitatur, et *membra animalium ad voluntatem moventur: vibrationibus ſcilicèt hujus ſpiritûs, per ſolida nervorum capillamenta ab externis ſenſuum organis ad cerebrum, et à cerebro in muſculos, propagatis*."

149. Fuſiùs hæc exponit, *Optical Quæries*:

"Is not *animal motion* performed by the power of this [*ætherial*] *medium*, excited in the *brain* by *the power of the will*, and propagated from thence through the ſolid, pellucid, and uniform, *capillamenta* [or minute fibres] of the nerves into the muſcles for contracting and dilating them?"—Vide § 195.

[N. B.] "I ſuppoſe that the *capillamenta* of the nerves are each of them *ſolid* and *uniform*, that *the vibrating motion of the ætherial medium* may be propagated along them from one end to the other uniformly and without interruption (for obſtructions in the nerves create *palſies*): and that they may be ſufficiently uniform, I ſuppoſe them to be *pellucid* when viewed *ſingly*, though the reflexions in their cylindrical ſurfaces may make the *whole nerve*, compoſed of many capillamenta, appear opaque and white."

Cautiſſimè igitùr diſtinguit *Newtonus* his locis, inter *mechanicas* operationes fluidi hujúſce ætherei ſeu "*ſpiritûs ſubtiliſſimi*"—et *mentem* ipſam ſeu *voluntatem*, quæ háſce ſentit, et quodam modo nobis prorsùs incognito, dirigit.

150. Et quàm ſollicitè diſtinguit quoque inter modificationes corporum, et ſenſationes quas in mente excitant! in *Optics*, p. 108.

"If at any time I ſpeak of *light* and *rays* as coloured or endued with colours, I would be underſtood to ſpeak not philoſophically and properly, but groſsly, and according to ſuch conceptions as vulgar people, in ſeeing all theſe experiments, would be apt to frame. For the rays, to ſpeak properly, are not coloured: in them there is nothing elſe but *a certain power and diſpoſition to ſtir up a ſenſation of this or that colour*: for as SOUND in a bell or muſical ſtring, or other *ſounding body*, is nothing but a *trembling motion*; and in the *air* is nothing but

but that motion *propagated* from the object; and in the *ſenſorium* 'tis a *ſenſe* of that motion under the form of *ſound:* ſo COLOURS in the *object* are nothing but a *diſpoſition* to *reflect* this or that ſort of rays more copiouſly than the reſt; in the *rays*, they are nothing but their *diſpoſitions to propagate* this or that motion into the *ſenſorium*; and in the *ſenſorium*, they are ſenſations of thoſe motions under the forms of *colours*."

DE SENSORIO ET DE VI SENTIENDI.

151. QUANTAS tragedias excitavit hæc vox technica *ſenſorium!* Reclamant *Leibnitziani:* "*Newtonum* hìc habes reum confitentem, et vim ſentiendi omnem ex cauſis *mechanicis* ſolventem!"

Sed ipſum audiamus, de induſtriâ definientem hanc vocem, "*the place of ſenſation.*"

"Is not the *ſenſory* of animals that *place* to which the ſenſitive ſubſtance is preſent, and into which the ſenſible ſpecies of things are carried through the nerves and brain; that there they might be *perceived* by their immediate preſence to *that ſubſtance?*"

And this diſtinction between the *organs of ſenſation* and the *ſenſitive ſubſtance* itſelf was long ago noticed by *Socrates*, in the *Phædo* of *Plato*, thus ridiculing the *Mechanical Philoſophiſts* of his day:

"It is true indeed, that without *bones* and *nerves*, and ſuch other *organs* as I poſſeſs, it would be impoſſible to *act* as I thought fit; but to aſſert that I do what I do by their *agency*, and not from *choice* of the beſt, would be downright abſurdity, and a mode of argument truly ridiculous."—Thus nobly ſtating his decided choice, rather to remain, and ſubmit to the unjuſt ſentence of the *Athenian* Judges, than to ſave his life by eſcaping to *Megara*, and ſeeking an aſylum among the *Bœotians*.

But was this preference merely the reſult of *vibratiunculæ* of the brain?

De induſtriâ itèm monet *Newtonus*, ne perperàm intelligatur *conjectura* ſua ſagaciſſima de modo ſentiendi;—ſe nullatenùs *theoriam* proponere: *Princip. fine.*

"Sed hæc *paucis* exponi non poſſunt; neque adeſt *ſufficiens copia experimentorum* quibus *leges* actionum hujus *ſpiritûs* [*ætherei*] accuratè determinari et monſtrari debent."

152. Con-

152. Conjecturæ *Newtonianæ* de actionibus hujusce *spiritûs*, seu *fluidi electrici*, adstipulari fas sit, recitando phænomena quædam, quæ mihi ipsi obtigerunt abhinc circitèr decennio; necnon viro clarissimo *Maclaurin*, moribundo; quæ satìs confirmare videntur *existentiam talis spiritûs in ipsis cerebri penetralibus actù latentis, et functionibus animi rectoris peragendis apprimè forsàn necessarii*; intimo quodam et incomprehensibili nexu inter *volitiones* et *actiones* subsistente, mediantibus organis corporeis. Nam, secundùm *Locke*:

"It may seem probable, that the *constitution* of the body does *sometimes* influence *the memory*; since we *sometimes* find a *disease* quite strip the mind of all it's *ideas*, and the flames of a *fever* in a few days calcine all those *images* to dust and confusion, which seemed to be as lasting, as if graved in marble."—*Essay*, *Book* II. c. 10.—See also § 195.

153. Æstate anni 1789, ipse in *febrem biliosam* incidi, ex diutinis laboribus, assiduis studiis, nimiísque curis contractam. Et quùm febriens insomnio miserè laborarem, medicus quidam ex peritioribus, mihíque amicus, *laudani* liquidi potionem administrabat, ad reducendum somnum. At spem prorsùs fefellit: licèt enim potio bìs térve repeteretur, manebat insomnium; et, quod notatu dignissimum est, unaquâque vice, decèm circitèr minutis temporis post potionem haustam, phænomenon singulare occurrebat: *quasi scintillæ ignis affatìm effluxerunt ex oculis meis, velutì ictu percitis.* Me ipso igitùr deprecante, intermissa est potio.—Exinde autèm valetudine ægerrimâ afflictus sum; affectionibus nervosis, et capitis lento dolore potissimùm, per triennium; obstructis quasì viribus animi;—inutilis mihi, inutilis propinquis et amicis, inutilis reipublicæ, ετωσιον αχθος αρȣρης.

Tandèm in plenitudine temporis atque mœroris, ubi cernere erat indicia haùd obscura *paralyseos* jamjàm imminentis, opitulatus est MEDICUS SUMMUS—Ἡ γαρ ΔΥΝΑΜΙΣ αυτȣ εν ΑΣΘΕΝΕΙΑ τελειȣται.—Mihi enìm suggessit ILLE DEUS, ut ex desidiâ ferè continuâ me extricarem, et ambulando strenuè me exercerem ad sudorem usque, si quâ ratione cieretur defluxio à cerebro, quod onere quasi incumbente valdè erat oppressum; et longè ultrà expectationem fuit successus: nam in fine hebdomadis unius tali exercitio occupatæ, sex aut septem horis per diem, defluxio humoris lymphatici, seu aquei, subitò erupit, cum admodùm incalescerem ambulando, et quasi à vertice ipso onus incumbens sublatum est, humore per glandes maxillares abundè stillante in os; et in fine hebdomadis alterius, gradatìm exsiccatâ defluxione, ad perfectam integrámque valetudinem restitutus sum; et meliorem quidèm quam per viginti antè annos. Diffugêre febres; redii ad me-ipsum, redii ad studia diù intermissa, redditus sum propinquis amicísque, redditus patriæ—Iterùm mihi concessa est, per misericordiam divinam, *Mens sana in corpore sano.*

DEO OPTIMO MAXIMO LAUS! *Luc.* xvii. 18.

154. Re-

154. Recolebam febriens iſto tempore, hujuſmodi phænomenon, vel eidem valdè ſimile, viro clariſſimo *Maclaurin* obtigiſſe moribundo; et inde erat quod averſatus atque deprecatus ſum, quaſi infauſtum et malè ominatum foret, potionem illam *laudani*, quæ id manifeſtò excitaverat; vaporibus forsàn laudani, (ut poſteà ſuſpicatus ſum), à ſtomacho calido diſſolutis, (qui, utpote valdè bilioſus, actionem laudani tunc reſpuebat,) et inde aſcendentibus in cerebrum; et *ignem electricum* ibi latentem expellentibus per nervos opticos; vaporibus ipſis in aquam poſteà converſis.

Caſus *Maclaurini* ità deſcribitur à biographo optimo:

—"His behaviour, during this tedious and painful diſtemper—[a *dropſy of the belly*, for which he had been thrice tapped]—was ſuch as became a PHILOSOPHER and a CHRISTIAN, calm, chearful and reſigned: his *ſenſes* and *judgment* remaining in their full vigour, till within a few hours of his death. Then, for the firſt time, his *Amanuenſis*, to whom he was dictating the laſt Chapter of the following work, [An Account of Sir *Iſaac Newton*'s Philoſophical Diſcoveries,] (in which he proves the wiſdom, the power, goodneſs, and other attributes of the DEITY,) obſerved ſome heſitation or repetition: no *pulſe* could then be felt in any part of his body; and his hands and feet were already cold. Notwithſtanding this extremely weak condition, he ſate in his chair, and ſpoke to his friend Dr. *Munro*, with his uſual ſerenity and ſtrength of reaſon; deſiring the Doctor to account for a *phænomenon* which he then obſerved in himſelf; *flaſhes of fire ſeemed to dart from his eyes; while, in the mean time, his ſight was failing, ſo that he ſcarce could diſtinguiſh one object from another.*—In a little time after this converſation, he deſired to be laid upon his bed; where, on *Saturday* the 14th of *June*, 1746, aged 48 years and four months, he had an *eaſy paſſage* (*) *from this world to that ſtate of bliſs*, which he had the moſt elevated ideas of, and which he moſt ardently longed to poſſeſs."

"*Let* ME *die the death of* THE RIGHTEOUS;
And let MY *laſt end be like* HIS!!"

(*) —Μεταβεβηκεν εκ τȣ θανατȣ εις την ζωην. *John*, v. 24.

"*Migravit* ex morte in vitam."

"Neque enim aſſentior iis qui hæc *nuper* diſſerere cœperunt," [*Lucretius*, &c.] "*Cum corporibus animos ſimùl interire, et omnia morte deleri.*" *Cicero de Amicitiâ.*

"M. Video te *altè ſpectare* et *velle in cœlum migrare.*
A. *Spero fore ut contingat id nobis.*" *Cicero, Tuſcul.* I.

Mac-

Maclaurin's Laſt Words.

155. The concluding, unfiniſhed paragraph of that ſublime Diſcourſe is too curious in the *Hiſtory of Man*, too precious to the *Chriſtian Philoſopher*, to be withheld on this occaſion :—

—" As Man is undoubtedly the *chief being* upon this globe ; and *this globe* may be no leſs conſiderable in the moſt valuable reſpects than any other in the *ſolar* ſyſtem ; and *this ſyſtem*, for aught we know, not inferior to any in the universal system : So, if we ſuppoſe *Man* to *periſh*, without ever arriving at a more competent knowledge of Nature, than the *very imperfect* one he attains in his *preſent* ſtate, by *analogy* (*d*) (or parity of reaſon), we might conclude, that the *like deſires* would be fruſtrated in the *inhabitants* of all the other *planets* and *ſyſtems* ; and that the *beautiful ſcheme* of Nature would never be unfolded, but in an exceedingly imperfect manner, to *any* of them :—This, therefore, naturally leads us to conſider our *preſent* ſtate,—as only *the dawn* or *beginning of our exiſtence* ; and as a ſtate of *preparation* or *probation*, for farther advancement : which appears to have been the opinion of the *moſt judicious Philoſophers* of old. [See *Heb.* xi. 13—16. and Note (*e*).]

" And

(*d*) It was a maxim, recorded of *Hermes*, and aſcribed to the primitive *Chaldæan* or *Perſian Magi* (the eſtabliſhed prieſthood of the Babyloniſh and Perſian Empire)—

Συμπαθη ειναι τα ανω τοις κατω.

" *That what paſſes in the heavens above is analogous to what paſſes on earth below.*"

But this ſage maxim, like many others, derived from *patriarchal* wiſdom, in proceſs of time was much abuſed by ſucceeding corrupt *poets* and degenerate *philoſophers*, receding from the native purity and ſimplicity of Revelation, and impiouſly attributing *human* paſſions to *celeſtial* beings. See § 169.

(*e*) The following curious and valuable anecdote we owe to Profeſſor *Robiſon*, p. 236 :—

" *Daniel Bernoulli*, the moſt elegant mathematician, the only philoſopher and the moſt worthy man of that celebrated family, ſaid to a gentleman (Dr. *Staehling*), who repeated it to me, that *when* reading ſome of thoſe *wonderful gueſſes* of Sir *Iſaac Newton*, the ſubſequent demonſtration of which has been the chief ſource of fame to his moſt celebrated commentators,—his *mind* has ſometimes been ſo overpowered by *thrilling emotions*, that he has wiſhed that moment to be his laſt ; and that it was *this* which gave him *the cleareſt conception of the happineſs of heaven*."

And ſhould not theſe " *thrilling emotions*," to which, I am perſuaded, Profeſſor *Robiſon* is no ſtranger, induce him to treat the Doctrine of a " *Vibrating Æther* " with more reſpect, ſo widely removed from " a *whim* made out of an *hypotheſis*," by *Newton*'s ingenuity ?—Should he not rather rank it among the moſt profound of his *wonderful gueſſes*,—approaching very nearly to the dignity of a *theory*, founded on ſuch *experiments* as the preceeding ?—How otherwiſe, are we to account for ſuch *thrilling emotions* of exſtatic delight ? ſuch *convulſive horrors* of agonizing fear ? ſuch ſtrange effects, as the change of the colour of the hair of a condemned criminal *from black* to *white* in the courſe of a ſingle night ? ſuch fatal and deadly diſruptions of the *material* vehicle, as are produced by ſudden

and

" And whoever attentively confiders THE CONSTITUTION OF HUMAM NATURE, particularly the *defires* and *paffions* of men, (which are *greatly fuperior* to their prefent objects,) will eafily be perfuaded, that MAN was *defigned* for *higher* views than thofe of this life: thefe, the AUTHOR OF NATURE *may* have in referve; to be opened-up to us at *proper periods* of time, and after *due preparation*: furely it is in HIS power to grant us a much greater *improvement* of the faculties we already poffefs; or even to endow us with *new* faculties (of which at this time we have no idea) for penetrating farther into the SCHEME OF NATURE, and approaching nearer to HIMSELF—THE FIRST AND SUPREME CAUSE:—We know not how far it was proper or neceffary, that WE fhould not be let into knowledge *at once*, but fhould advance *gradually*; that, by comparing new objects or new difcoveries with what was known to us before, our improvement might be more compleat and regular: or how far it may be neceffary or advantageous that INTELLECTUAL BEINGS fhould pafs through a kind of *infancy of knowledge*; for new knowledge does not confift fo much in our having accefs to a new object, as in comparing it with others already known; obferving it's *relations* to them (or difcerning what it has in

and overpowering tranfports of joy or grief?—Thefe are notorious *effects*: and how they are to be accounted for, upon attentive confideration of the *conftitution of human nature*, compofed of a *mind* operating upon it's receptacle or *fenforium*, otherwife than by fome inconceivable *ftimulus* of the nervous fyftem, conveyed through *mufcular vibrations*, or fome *impulfe* analogous thereto, I leave to his ingenuity to fuggeft.

2. To the tafte and piety of Profeffor *Robifon* (whom I hail as a *Brother*, notwithftanding our difagreement in one inftance,) we are alfo indebted for circulating in his defervedly popular work, *Mafon*'s Sacred Ode, drawn from the pureft fources of the *Pierian Spring*:

" Think not THE MUSE whofe *fober* voice you hear,
Contracts with *bigot* frown her fullen brow;
Cafts round RELIGION's orb the mifts of *fear*,
Or fhades with horror, what with *fmiles* fhould glow:

No.—SHE would warm you with *feraphic fire*;—
Heirs as you are of HEAVEN's eternal day;—
Would bid you *boldly* to THAT HEAVEN *afpire*,
Not fink and flumber in yon cells of clay!—

Is this the *bigot*'s rant?—Away, *ye vain!*
Your doubts, your fears, in *gloomy dulnefs* fteep:
Go!—*foothe* your fouls in ficknefs, death, or pain,
With the fad folace of eternal fleep!

Yet know, *vain fceptic!* know, th' ALMIGHTY MIND,
Who breath'd on *Man* a portion of HIS fire;
Bade his *free foul*—by earth nor time confin'd—
To HEAVEN; to IMMORTALITY; afpire!

Nor fhall this *pile of hope*, HIS BOUNTY rear'd,
By *vain Philofophy* be e'er deftroyed;
ETERNITY, by ALL or *hoped* or *fear'd*,
Shall be by ALL or *fuffer'd* or *enjoy'd*."

common with them), and wherein their *diſparity* conſiſts. Thus, our knowledge is vaſtly greater than *the ſum* of what all it's objects ſeparately could afford; and when a *new* object comes within our reach, the addition to our knowledge is the greater, the more we already know: ſo that it increaſes, not *as* the new objects increaſe, but in a *much higher proportion.*" * * *

156. And was the exalted *mind*, that reaſoned thus powerfully, and triumphed over *bodily* infirmity in the very jaws of *death*, *material?*—Was all this merely the reſult of corporeal *mechaniſm?* Was the profound proceſs of this noble argument for the ſurvival and improvement of the ſoul; the grand *climax* at the beginning, riſing from this globe to the milky way; ſuch clear-ſighted intelligence of the maſter principles and lofty affections, and aſpiring views of *human nature*, in the ſequel; ſuch ſubtile and abſtruſe analyſis of the doctrine of *combinations* and of the *fluxionary* calculus, applied at it's unfiniſhed cloſe; ſuch humble, yet aſtoniſhing penetration and inſight into the myſterious ways of Providence, in this life—"*Looking thro'* NATURE *up to Nature's* GOD:"—while the quivering lamp of light, thus darting the brighteſt flaſhes of celeſtial fire, was expiring in the ſocket.—Was all this admirable aſſemblage, of the pious and the profound, the ſublime and the beautiful, the argumentative and the eloquent, no other, ye *Hartleys!* ye *Prieſtleys!* ye *Condorcets!* ye *Godwins!* or ye *Darwins!* than mere animal mechaniſm? wrought and effected, wholly and ſolely, by *vibratiunculæ* or minute tremors of the *pulpy* ſubſtance of the brain!!! (*f*)

"O curvæ in terras animæ et cœleſtium inanes!
Quid juvat hoc, *templis* noſtros immittere *mores*;
Et *Bona* DIS ex hâc *ſceleratâ* ducere *pulpâ?*" PERSIUS.

D'Alem-

(*f*) The ſublime Philoſophy of HOLY WRIT, and that alone, has *authoritatively* decided the myſterious and momentous queſtions of the *immateriality* of the *Soul*, of it's natural *mortality*, and of it's *reunion* with an *incorruptible body* after death; recording the *Origin* and the *End* of the *Human Race*:—

1. "THE LORD OF GODS formed *the Man*, *duſt* of the ground,
And breathed into his noſtrils, *breath of life*;
So *Man* became a *living ſoul.*" *Gen.* ii. 7.

2. "*Duſt* thou art, and unto *duſt* ſhalt thou return." *Gen.* iii. 19.

3. "All *Fleſh* ſhall periſh together;
And *Man* ſhall turn again unto *duſt.*" *Job* xxxiv. 15.

4. "I know that my DELIVERER is LIVING;
And at the *laſt* [*day*] will ariſe [in judgment] upon *duſt:*
Yet, after this *ſkin* be mangled,
Even in my *fleſh* ſhall I perceive GOD;
Whom I ſhall perceive, *on my ſide*,
And mine eyes ſhall ſee [HIM] *not eſtranged.*" *Job* xix. 25.

5. "The

D'Alembert's Mournful Reflections.

157. As a terrific contraſt to the death of the *Chriſtian Philoſopher*, I ſhall propoſe the dreadful gloom that overhung the latter days of a *philoſophizing infidel*, furniſhed by his correſpondence with the King of *Pruſſia*, another of the original conſpirators againſt Chriſtianity in the train of *Voltaire*.—*D'Alembert* —at once the glory and diſgrace of the French *Academy of Sciences*—"*deceiving*" others, by his perfidious ſophiſtry, but not fully "*deceived*" himſelf—ſo as to conquer and blunt the ſtings of remorſe, and the terrors of a guilty conſcience, anticipating "the juſt reward" of thoſe fiends in human form, who,

5. "The *duſt* ſhall return to *the earth* as it *was*;
But *the ſpirit* ſhall return to God who gave it." *Eccleſ.* xii. 7.

6. "Jesus Christ—"illuſtrated *life* and *incorruption*." 2 *Tim.* i. 10.
"Himſelf—"*the firſt fruits* of the *ſleepers*." 1 *Cor.* xv. 20.

7. "Lo! I tell you a *myſtery*:
We ſhall not all *ſleep*, but we ſhall all be *changed*;
In a moment, in the twinkling of an eye,
At the *laſt trumpet*."——

8. "For this *corruptible* (body) muſt put on *incorruption*;
And this *mortal* (ſpirit) muſt put on *immortality*."

9. "O *Death!* where is thy *ſting* [in the *body*]?
O *Hades!* where is thy *victory* [over the *ſoul*]?

10. "The *ſting* of *Death* is *Sin*———
But thanks be to God, who giveth us the *victory*,
Through our Lord Jesus Christ." 1 *Cor.* xv. 51—57.

11. "I am the *Firſt* and the *Laſt* and the *Living*,
And I became *dead*; and lo! I am *living*
For ever and ever. Amen.
And I hold the keys of *Hades* and of *Death*." *Rev.* i. 17.

12. "I ſay unto you, *my friends*:
Be not afraid of them who kill the *body*,
But are not able to kill the *ſoul*;
But be afraid rather of Him, who is able
To deſtroy both *ſoul* and *body* in *hell*." *Matt.* x. 28.

13. "Who will transform the *body of an humiliation*,
To become the reſemblance of the *body of his glory*;
According to the energy of his ability,
Even *to ſubject all things unto himſelf*." *Phil.* iii. 21.

who, after zealoufly labouring through life to undermine the *hopes* of others, too fatally fucceed at length in blafting their own!

In one of his laft letters, publifhed by the ingenious, the unprincipled, the execrable, and the wretched, *Condorcet*; who befet *D'Alembert* when expiring, and boafted, that, if he had not, "*D'Alembert would have flinched alfo*,"—like their grand mafter *Voltaire*—in all his dying horrors and agonies, "*worfe than the furies of Oreftes*"—the peevifh and gloomy philofophift thus complains to his royal correfpondent, *Frederick:*

—"Study *fometimes* engages me, and converfation *fometimes* entertains me; but I am foon fatigued with either; and am no fooner left to myfelf than my *uneafy reflections* recur, and my *folitude* again frightens and freezes me. In this condition, I refemble a man who fees before him *a long and dreary defart*, which he *muft* pafs; and at the end of that *melancholy profpect*, the *abyfs of deftruction* open to receive him: without finding, at the brink of that *hideous chafm*, a fingle perfon that will be *afflicted* with his downfal, or that will even *remember* his exiftence, when he has funk into *endlefs perdition!!!*"

From SUCH *Philofophy*, GOOD LORD! *deliver us.*

158. How utterly falfe, then, and unfounded are the vifionary fpeculations of the votaries of "*a miftrefs that never grows grey!*" (if we liften to the illi-

14. "For *flefh* and *blood* [i. e. an *earthly* body] is not able
To inherit GOD's kingdom; neither fhall *corruption*
Inherit *incorruption*." 1 *Cor.* xv. 50.

15. "The mafs of *the fleepers*, [though] *duft of the earth*,
Shall awake; fome to everlafting *life*,
But fome to fhame and everlafting *contempt*:
When the *fages* fhall fhine, as the *brightnefs*
Of the firmament; and the *juftified* of the many,
As the *ftars* for evermore." *Dan.* xii. 2.

16. "As *one* ftar differeth from *another* ftar
In *glory*." 1 *Cor.* xv. 41.

17. "In my FATHER's houfe are *many manfions*." *John* xiv. 2.

Such are the awful, the animating, the ennobling doctrines of *Patriarchal* and *Evangelical* WISDOM—*revealed*, "at fundry times and in divers manners," to his *rational* creatures, by the FATHER OF LIGHTS, through 'THE SON OF HIS LOVE'—THE ORACLE or Expounder of his Decrees, under the *Patriarchal* and *Chriftian Economy*. Nor can this important felection of ftrictly harmonizing *Texts* be deemed irrelevant or unfeafonable by the *Mathematician* or the *Philofopher*, who would wifh zealoufly to ftem that Torrent of *Infidelity*, which is the difgrace of a boafted AGE OF REASON:

Difcite, O miferi! et *caufas* cognofcite rerum:
Quid *fumus*, et quidnam *victuri* gignimur; *Ordo*
Quis datus;——quem *Te* DEUS effe
Juffit.—— PERSIUS.

terate

terate *Paine*'s eulogy on SCIENCE)—And how wretched, how delusive the *happiness* she can afford, detached from RELIGION—that true mistress of the *wise* and *good*—whose humble handmaid SCIENCE ought to be.

—" *To be happy in old age*," says *Paine*, " it is necessary that we accustom ourselves to objects that *can* accompany the mind *all the way through life*; and that we take the rest as good in their day. The mere man of *pleasure* is miserable in old age; and the mere drudge in *business* is but little better: whereas *Natural Philosophy*, *Mathematical* and *Mechanical Sciences*, are a continual source of *tranquil pleasure*. And, in spite of the gloomy dogmas of *Priests* and of *Superstition*, the study of these things is the study of the *True Theology*; it teaches man to *know* and *admire* the CREATOR: for the *principles of Science* are in the *Creation*, are *unchangeable* and of *divine origin*."

159. Alas! what a wretched happiness did *Science* procure *D'Alembert*, who, at the close of his days, thus found to his cost, that he had been all his life long embracing a *cloud* for a *goddess*!—And that "the study of *these things* is not indeed the study of the *True Theology*," we learn from the monstrous *errors* of ancient and modern *Sophists*; which led them to *forget* and to *vilify* the CREATOR: " The *principles of Science* are *not* indeed in the Creation;"—they lie much deeper than the *visible* world; and are only to be learned from REVEALED WISDOM and OMNISCIENCE:—whose *Sovereign Will* is the *Law* of the Universe; *changeable* at pleasure, though in HIMSELF "*unchangeable*." *James*, i. 17.

160. These principles, the lowly and unassuming spirit of *Primæval* and *Patriarchal Science* endeavoured cautiously to trace and develope, amidst " the *obscurity* that prevails in the midst of things;"—humbly and devoutly thanking the FATHER OF LIGHTS for those faint *glimpses* of ETERNAL TRUTHS, which, from time to time, and from season to season, He vouchsafes to reveal to those truly Philosophic Students, who, in " an *honest and good heart*," *hear*, seek, embrace, and keep the *Word of God*, whether communicated by *Revelation*, or thence unfolded by *Reason*: and earnestly and diligently, and devoutly, strive to *bring forth fruit with patience*.

" *Cœlum* quippè ac *Terra*, ac *Mare*, omnísque *Creatura* quæ videri atque intelligi potest, ad hanc præcipuè dispofita est *humani generis* utilitatem, ut *Natura Rationalis* de contemplatione tot specierum, de experimentis tot bonorum, de perceptione tot munerum, ad cultum et dilectionem SUI imbueretur AUCTORIS; *implente omnia* SPIRITU DEI, "*in quo vivimus, movemur, et sumus*."—" Adhibita semper est universis hominibus *quædam* supernæ mensura doctrinæ, quæ [licet] *parcioris occultiorisque gratiæ*, sufficit tamèn (sicùt DOMINUS judicavit), quibusdam ad remedium, omnibus ad *testimonium*.—Ait *Plotinus*: " *Si* MUNDI *vocem audiremus*, nihil aliùd eum dicere quam DEUS ME FECIT; *non Cretensis Jupiter aut Arcas Mercurius, sed* DEUS *ille* ΑΓΝΩΣΤΟΣ [*nescibilis*], de quo *Paulus* ad *Athenienses*." *De Vocatione Gentilium.*

Vide posthàc, § 205, Not. (*p*).

Scriptori

Scriptori huic egregio, veriffimè de Dei cognitione, quatenùs ex rationibus *phyficis* eam colligere fas fit, fuffragatur fana *Newtoni* fententia: *Princip.* p. 529.

" *De* DEO utique ex *phænomenis* differere ad PHILOSOPHIAM NATURALEM *pertinet.*"

161. Placet igitùr, quoniam omnis difquifitio de *caufis* et *effectis* naturalibus ad agnitionem CAUSÆ PRIMARIÆ perducere debet, " quæ *minimè materialis* eft," *Newtono* judice — VINDICIAS quafdam THEOLOGICAS *Sapientum Prifcorum*, atque *Newtoni* horum cultoris, fubjungere.

De Philofophiâ Atomicâ Prifcorum Chaldæorum atque Hebræorum.

---

162. Nec minùs injuriosè invehitur *Robifon* in *Philofophiam Atomicam*, qualem excoluit *Newtonus*; quæ minimè confundenda eft cum *Epicureâ* iftâ nupèr redivivâ, Rationibus Metaphyficis atque Mechanicis rerum naturalium caufas tribuente, et CAUSÆ PRIMARIÆ oblitâ. Hanc enim jure perftringit ipfe *Newtonus* in *Optics*, p. 343, his verbis:

" And for rejecting fuch a *medium* (as is totally denfe or full of matter), we have the authority of the oldeft and moft celebrated Philofophers of *Greece* and *Phœnicia*, who made a *vacuum* and *atoms* and the *gravity* of atoms the firft principles of their philofophy: tacitly attributing gravity to *fome other caufe* than denfe matter."

" Later Philofophers banifh the confideration of *fuch a caufe* out of *Natural Philofophy*; feigning *hypothefes* for explaining all things *mechanically*, and referring other caufes to Metaphyfics: whereas the main bufinefs of Natural Philofophy is to argue from *phænomena*, without feigning hypothefes, and to deduce *caufes* from effects, till we come to the VERY FIRST CAUSE, which certainly is not *material*."

Hæc breviter effata et delibata tantùm, fufiùs exponere, atque ex philofophiâ primâ confirmare, jam libet:

163. Prifcorum Chaldæorum atque Hebræorum, Philofophia phyfica erat *Atomica*, Cofmogoniæ *Mofaicæ* confentanea. Hanc invexerunt in Thraciam, *Orpheus*, ille " *Sacer interprêfque Deorum*," qui undecim generationes antè Trojæ excidium floruit, tefte *Suidâ*, feu annis 366 + 1184 = 1550 circitèr antè Æram Chrifti Vulgarem; in Ioniam et Græciam, *Thales*, " *phyficorum princeps*," *Pherecydis* Syri difcipulus; in Italiam, *Pythagoras.*—Lucida fidera omnes.

Quanto

Quanto fuerunt in honore Sapientes *Chaldæi* et *Hebræi*, apud *Græcos* vetuſtiores, diſcimus ex reſponſo *Oraculi Pythici*, quibuſdam ſciſcitantibus, *Ex gentibus quinam omnium ſapientiſſimi?*

Μȣνοι Καλδαιοι σοφιην λαχον, ηδ' αρ' Ἑβραιοι,
Αυτογενητον Ανακ]α σεϐαζομενοι ΘΕΟΝ ΑΥΤΟΝ.

"Soli *Chaldæi* ſapientiam ſortiti ſunt atque *Hebræi*,
Per-ſe-genitum Regem venerantes Deum ipsum."

164. Præclariſſimum extat teſtimonium in fragmentis *Verſuum Orphicorum*, ab *Onomacrito* congeſtis, de Decalogo Sacro, apud Montem *Sinai* promulgato, annis 1649 circitèr ante *Ær. Chriſt.* (ſecundùm emendationem noſtram *Sacræ Chronologiæ*, poſthàc forsàn edendam).

Αρχην ΑΥΤΟΣ εχων, ἁμα και Μεσον, ηδε Τελευ]ην·
Ὡς λογ⊛ Αρχαιων· ὡς Ὑδογενης διε]αξεν,
Εκ ΘΕΟΘΕΝ γνώμαισι λαϐων κα]α διπλακα Θεσμον.

"*Principium* Ipse habens, ſimùl et *Medium*, atque *Finem*:
Ut ſermo Priſcorum; ut *Aqua-genitus* præcepit,
E Deo ſententiis quum accepiſſet *duplicem Legem*."

Præclara hæc deſcriptio Dei Ipsius optimè conſentit cum *Scripturis Sacris*. *Iſa.* xli. 4. *Rev.* i. 8. &c. &c.

"Him *Firſt*, Him *Laſt*, Him *Midſt*, and *without End*." *Milton*.

Vox Ὑδ⊛, antiquitùs pro Ὑδωρ, "*aqua*."—*Heſychius*. Per Ὑδογενης igitùr elegantèr expreſſit *Orpheus* vocem Hebræam *Moſeh* (vel *Moſes*), quæ ſignificat "ex aquâ *ereptum*."—*Exod.* ii. 10. Et per διπλακα Θεσμον, "duplicem *Decalogi* Tabellam"—"digito Dei deſcriptam." *Exod.* xxxii. 26. et xxxiv. 1. (*g*). Atque hoc *Orphei* teſtimonium expreſſiſſe videtur *Juvenalis*, *Sat.* xiv. 102.

"Tradidit arcano quodcunque volumine *Moſes*."

165. Alterum

(*g*) Hujúſce eximii *Fragmenti Orphici*, ab *Onomacrito* conſervati, auctoritatem inſectatur doctiſſimus *Cudworth*, ὁ πλατωνιζων, ſed hypercriticè potiùs, ut videtur, *Intellectual Syſtem*, p. 300.

"There ſeem to be ſome *Orphic* Verſes ſuppoſititious, as well as there were *Sibylline*, &c.—Cæterùm cum hiſce probè conſentiunt verſus *Orphici* quos ipſe *Cudworth* pro genuinis agnoſcit, et ex *Proclo* citat, p. 301.

ΖΕΥΣ πρωτος γενε]ο, ΖΕΥΣ ὑςατος αργικεραυνος,
ΖΕΥΣ κεφαλη, ΖΕΥΣ μεσσα, ΔΙΟΣ δ' εκ παν]α τετευκ]αι.

"Jovis *primus* exiſtebat, Jovis *ultimus* altinonans,
Jovis *caput*, Jovis *medius*, à Jove *omnia fabricantur*."

Et

165. Alterum, quoque, nec minùs insigne testimonium de *Chaldæorum* divinatione profert *Orpheus*.

Ου γαρ κεν τις ιδοι θνητων μεροπων ΚΡΑΙΝΟΝΤΑ,
Ει μη μȣνογενης τις, απορρωξ φυλȣ αωθεν
Χαλδαιων· Ιδρις γαρ εην Αστροιο πορειας.

" Nullatenùs enìm quisquam mortalium articulatè loquentium viderit REGEM,
Nisi *dilectus* quidam antiquo ab origine gentis
Chaldæorum; peritus enìm erat *astri exortûs*."

Referunt hoc vulgò interpretes ad *Abrahamum*, " quem omnis vetustas Orientis summum astrologum fuisse crediderit." *Scaliger* de *Emend. Temp.* Not. in *Fragment*. p. 48; et *Cudworth*, p. 300. Sed manifestò respicit vatem Chaldæum, *Balaam*, *Mosis* co-ævum, " qui apud *Peth-Ur* (seu Hebräicè *Beth-Ur*, " Domus Lucis,") juxtà fluvium *(Tigrim)* degebat," (*Numb*. xxii. 5.); et qui insigne illud vaticinium cecinit de *Shilo*, *Jacobi* progenie, (*Gen*. xlix. 10.) seu de *Messiâ* " Rege," *Num*. xxiv. 17.

" *Video* ILLUM, *sed non nunc*;
Cerno ILLUM, *sed non propè*;
Prodibit ASTRUM *ex Jacobo*,
Exorietur SCEPTRUM *ex Isräele*."

Cujúsque posteri, Μαγοι απο ανατολων, " *Magi Orientales*,"—actu " *viderunt astrum ejus in exortu suo*," (εν τη ανατολη,) 1600 annis posteà, è longinquo conspicati, *nocte* forsàn *natali* Christi. Confer *Matt*. ii. 1—10, et *Luc*. ii. 9—15.

166. Priscos *Chaldæos* atque *Hebræos vacuum* et *atomas* admisisse in cosmogoniam suam, colligere licet ex *Virgilio*, mythologiæ antiquæ observantissimo,

Et versum secundum, levi variatione, ex ipso *Plutarcho* quoque citat, p. 305.

ΖΕΥΣ αρχη, ΖΕΥΣ μεσσα, ΔΙΟΣ δ' εκ παντα πελονται.
" JOVIS *principium*, JOVIS *medius*, à JOVE *omnia fiunt*."

Ubi pro κεφαλη citantis *Procli*, *Plutarchus* vocem synonymam αρχη citat; utramque proculdubio à voce *Hebræâ*, ראש, vel ראשית ambiguè " *caput*" vel " *principium*" designante. *Gen*. i. 1.—Vide quoque Not. (*n*).

Necnon ex *Timotheo* chronographo quasi *Orphicæ Theologiæ* foret; p. 305.

Δια της ΘΕΟΤΗΤΟΣ παντα εγενετο,
Και ΑΥΤΟΣ εστι παντα.

" Per DEITATEM omnia facta sunt,
Et IPSE est Omnia."

Veruntamèn fatendum est quòd *Orpheus* ipse falsa veris immiscuit. Refert enim *Justinus Martyr*; Πολυθεοτητος πατηρ και πρωτος διδασκαλος—quòd " *Polytheismi pater atque præceptor primus fuit*" *Orpheus*. Vide *Cudworth*, Int. Syst. p. 298.

ubi *vinoſum Silenum*—" *Cuſtodem famulúmque Dei (Bacchi) alumni*," et proinde ex Patriarchis forsàn vetuſtiſſimum, (inſtar *Noah*, vino oppreſſi, *Gen.* ix. 21—24.) ità vaticinantem inducit, *Eclog.* vi. 31.

" Námque canebat, utì MAGNUM per INANE coacta
Semina *terrarúmque*, *animaéque*; *maríſque* fuiſſent,
Et *liquidi* ſimùl *ignis*: ut his exordia primis
Omnia; et ipſe tener Mundi *concreverit* orbis."

167. Paritérque *Euripides*, *Anaxagoræ* diſcipulus, traditionem vetuſtiſſimam refert, in fragmento *Menalippes*, *Barnes Eurip.* p. 481, citato à *Diodoro Siculo*, I.

Ουκ εμος ὁ μυθος, αλλ' εμης μητρος παρα·
Ὡς ουρανος τε γαια τ' ην μορφη μια·
Επει δ' εχωρισθησαν αλληλων διχα,
Τικτουσι παντα, κ' ανεδωκαν εις φαος,
Δενδρα, πτηνα, θηρας, ὅς θ' ἁλμη θρεφει,
Γενος τε θνητων.

" Nec meus hic ſermo, ſed quem præcepit Mater:—
Ut cœlum et terra erant forma una;
Ubi verò à ſe invicèm ſejuncta ſunt,
Pariunt omnia, ediderúntque in lucem,
Arbores, *aves*, *feras*, [*piſcéſque*] quos alit mare,
Genúſque mortalium."

Manifeſtò deſumuntur hæc *mythica* ex Coſmogoniâ *Moſaicâ*; de ſtatu terræ *chaotico*, et generatione *vegetabilium*, *animalium* atque *hominum*, ordine *Moſaico* differentia; cæterùm in hoc peccant quod SUMMI OPIFICIS vim creatricem vix, et ne vix quidem, reſpiciunt, ſed productionem rerum *ſpontaneam* innuunt.

168. Atqui híſce excerptis, ſeminarium habes *Philoſophiæ Atomicæ* qualem excolebant Sapientes Scholæ *Chaldææ primævæ* ac *Hebrææ*, antequàm eam peſſundederunt atque inquinaverunt pravè philoſophantes, *Magi*, *Brachmanni*, &c. Orientales; atque *Stoici*, *Epicuræi*, &c. Occidentales, ingenti veræ Theologiæ atque Philoſophiæ detrimento.

169. Et egregiè diſtinguit *Plutarchus*, inter " *valdè antiquos theologos atque poëtas*"—quales fuerunt *Orpheus* ejúſque diſcipuli, qui "JOVEM *principium*, JOVEM *medium*, à JOVE *omnia fiunt*"—præceperunt, et " *híſce juniores, qui Phyſici ſunt appellati*"—quales poſt *Thaletem* exſtiterunt *Anaximander*, *Anaximenes*, &c. Scholæ *Ionicæ*; atque *Itali* iſti degeneres Scholæ *Pythagoreæ*, *Leucippus*, *Democritus* et *Epicurus*, qui, " *ab hoc principio honeſto atque divino temerè palantes, cauſam*

omnium

omnium primariam in corporibus, corporeísque affectionibus, pulsibus, mutationibus commixtionibúsque posuerunt." — Undè Φυσικοι et Αθεοι ("*Naturalists*" and "*Atheists*") indifferentèr, et quasi per nomina ejusdem ferè significationis, vulgò sunt appellati. *Cudw.* p. 305.

170. Etiàm *Anaxagoras* ipse, appellatus Νȣς, "*Mens,*" κατ'-εξοχην, qui priscam notionem MENTIS SUPREMÆ revocavit, "Ejus opificis in mundo condendo, rationibúsque physicis, nullatenùs est usus, sed ad causas rerum merè mechanicas, *aëres, ætheres*, et *aquas* minimè idoneas, aliáque prorsùs absurda, confugit," ità querente *Socrate*, in *Phædone*.

171. Hinc ansam vituperandi arripiens *Aristoteles*, totam *Physicam Atomicam* insectabatur præceps, ex rationibus suis *Metaphysicis*, multa somnians de *quinto genere elementorum è quo fiant mentes et astra*, de *fugâ vacui*, de *pleno universali*, de *æternitate mundi*, de *formis substantialibus* (seu εντελεχειαις). Inanitèr autèm atque arrogantèr gloriatur—"*Se videre, quod paucis annis magna accessio facta esset; brevi tempore, Philosophiam planè absolutam fore.*"—*Cicero*, *Tuscul.* I. 10; III. 28. *Acad.* I. 4.

172. *Aristotelis* fastum jactantiámque secuti, *Cartesiani* et *Leibnitziani*, ingenti scientiarum detrimento, Physicorum Græcorum *Mechanicam* irrationalem et phantasticam recolebant; *vortices, monadas*, &c. temerè fingentes: donec modestum *Newtoni* ingenium priscas revocabat artes; atque à Logomachiis aversus, ad Disciplinam Scholæ *Ionicæ*, *Antiquæ Hebrææ* atque *Chaldææ*, serò tandèm redibat, Physicam *Atomicam* instaurabat, eam fundamentis solidis atque inconcussis *Inductionis* atque *Experimentorum* stabiliendo.

—"*Hypotheses* non fingo: Quicquid enìm ex *phænomenis* non deducitur, Hypothesis vocanda est. Et Hypotheses, seu *Metaphysicæ*, seu *Physicæ*, seu *Qualitatum occultarum*, seu *Mechanicæ*, in PHILOSOPHIA EXPERIMENTALI locum non habent." *Principia, pag. ult.*

173. Quàm injuriosè igitùr invehitur Professor *Robison* in *Philosophiam Atomicam*, qualem excolebant prisci *Chaldæi* et *Hebræi*, eorúmque Discipuli; et qualem instaurabat *Newtonus*, sobriam, sanam, *principiis Mathematicis* reconditioribus et subtilioribus forsàn quam pro captu tyronum, imò Mathematicè doctorum, (etiàm *Professorum* forsàn), stabilitam atque demonstratam. *Logicam* certè parùm sapit, *ex abusu* alicujus rei, artis vel scientiæ, *usum* detrectare; et *Mechanismum* adeò irrationalem, absurdum et impium Recentiorum Philosophistarum, ex *Mechanicâ Rationali Newtonianâ* deducere.

De

De Æthere Antiquo, Deo Ficto Gentilium.

---

174. Vocem ipſam *Æther* mutuatus eſt *Newtonus* ex Philoſophiâ Priſcâ, ità referente *Cicerone*. N. D. II. 33.

" Et cùm quatuor ſint genera corporum, *viciſſitudine* eorum mundi continuata natura eſt. Nam ex *terrâ*, aqua; ex *aquâ* oritur aër; ex *aëre*, *æther:* Dein retrorsùm viciſsìm, ex *æthere*, aër; ex *aëre*, aqua; ex *aquâ*, *terra infima*, &c."

Exemplum hìc habes *Compoſitionis* et *Reſolutionis* antiquiſſimæ, ſecundùm quas " *mundi partium conjunctio continetur* "—ex mente Phyſicorum priſcorum, ſanum, Cantui *Sileni* atque *Coſmogoniæ Moſaicæ* accommodatiſſimum: ubì *æther* manifeſtò reſpondet " *liquido igni*," quarto nimirùm elemento, (hodiè, *fluido electrico*) ſubtiliſſimo, omnium corporum poros et interſtitia liberrimè permeanti, quod inter potentiſſimos naturæ agentes jure recenſetur; et cujus identitatem cum *igne cœleſti (lightning)* detexit et confirmavit, noſtro tandèm ævo, curioſa ſedulitas ſagaciſſimi Naturæ indagatoris, *Benjamini Franklin*.

Vox αιϑηρ quoque, ſi compoſitionem ejus reſpicias, derivatur ex αιϑω, " *uro*," et αηρ, " *aër*," ſeu *atmoſphæra* (quæ forsàn ex אור *aur*, " *lux*," ultimò deducenda eſt). Et quidèm ab *Heſychio* definitur Αιϑηρ, ὁ ὑπερ τα νεφη τοπος, " *regio ſuprà nubes*;"—atque ſignificatione alterâ ſynonymâ, εμπυρισμον, " *inflammationem*."

175. Pro ſymbolo (*b*) certè " Dei *lucem inacceſſibilem habitantis* " primitùs, Patriarcharum ſæculis, *Job*, *Abraham* atque " *Chaldæorum* progenitorum, *Aur*, (i. e.

(*b*) Exordium Elegiæ à reverendo viro et poetâ docto et diſerto, *Thomâ Maurice* ſcriptæ in obitum celeberrimi et optimi viri, *Gulielmi Jones*, Indorum Apoſtoli, Chriſtianique Philoſophi, pulchrè et ornatè deſcribit patrias ſedes *Chaldæorum*, et primævam diſciplinam *Magorum*.

" Where the dark cliffs of rugged *Taurus* riſe,
From age to age by blaſting lightnings torn,
In glory burſting from the illumin'd ſkies,
Fair *Science* pour'd her firſt auſpicious morn.

The hoary *Parthian* Seers, who watch'd by night
Th' eternal Fire in *Mithra*'s myſtic cave,
(*Emblem* ſublime of that PRIMÆVAL LIGHT
Which to yon ſtarry orbs their luſtre gave,)

Exulting, ſaw it's gradual ſplendours break,
And ſwept, ſymphonious, all their warbling lyres,
'Mid *Scythia*'s frozen glooms THE MUSES wake,
While happier *India* glows with all their fires."

Et

(i. e. "*Lux*,") incolentium," *Gen.* xi. 28. usurpatus est Αιθηρ, seu "*Aër* ille *igneus*."

176. *Theologiæ Chaldææ* præclarissimum extat specimen, apùd *Suidam*, de *Orpheo* ità differentem :

Εφησε δε, ὁτι ΦΩΣ, *ῥηξαν τον* Αιθερα, *εφωτισε την* Γην, *και πασαν* Κτισιν· *Εκεινο ειπων* "ΤΟ ΦΩΣ" *το* 'ΥΠΕΡΤΑΤΟΝ *παντων, το* "ΑΠΡΟΣΙΤΟΝ," *το παντα* ΠΕΡΙΕΧΟΝ· *'Οπερ ωνομασε* ΒΟΥΛΗΝ, ΦΩΣ, ΖΩΗΝ.

Ταυτα τα τρια Ονοματα ΜΙΑΝ ΔΥΝΑΜΙΝ *απεφηνατο και* 'ΕΝ ΚΡΑΤΟΣ *τȣ* ΔΗΜΙΟΥΡΓΟΥ *παντων* ΘΕΟΥ, *τȣ παντα εκ τȣ μη οντος παραγαγοντος εις το ειναι, ὁρατα τε και αορατα.*

"Fatus est *Orpheus*, LUCEM, rupto *Æthere*, Tellurem illustrâsse, omnémque Creationem : Illam dicens LUCEM, *supremam* omnium, "*inaccessibilem*," omnia *continentem* ; quam nominavit CONSILIUM, LUCEM, VITAM."

"Hæc tria nomina declarant UNAM POTENTIAM, UNUMQUE DOMINIUM OPIFICIS *omnium* DEI, qui adduxit omnia *ex non existente in existentiam, visibilia paritèr et invisibilia*."

177. Mirus sanè et ferè incredibilis consensus hic observandus cum Libris Evangelicis —Nam "DEUS est LUX," 1 *John*, i. 5. "*Lucem habitans inaccessibilem*," 1 *Tim.* vi. 16. Et JESUS CHRISTUS est "LUX *Mundi*," *John* viii. 12. Ut et "SAPIENTIA DEI," *Luk.* vii. 35. et *Matt.* xi. 19.—itémque "VITA," *John* i. 4. et xiv. 16. Et declaravit CHRISTUS, *Εγω και Πατηρ* 'ΕΝ *εσμεν.*—*Orpheo* nimirùm optimè interpretante—'ΕΝ ΚΡΑΤΟΣ—" PATER et EGO *unum sumus* [DOMINIUM]." *John* x. 30.

Et *Milton*, adhùc sublimiùs et eruditiùs, varias sententias veterum Philosophorum exponit, in celeberrimâ illâ LUCIS invocatione, P. L. III. sub initio :

"Hail, HOLY LIGHT! Offspring of Heaven, First-born,
Or of th' eternal co-eternal Beam ;
May I express thee unblam'd! Since "GOD IS LIGHT,"
And never but in "*unapproached light*
Dwelt" from Eternity, dwelt then in Thee,
Bright *Effluence* of bright ESSENCE INCREATE :
Or hear'st thou, rather, pure *Ætherial Stream*,
Whose fountain who shall tell ? Before *the Sun*,
Before *the Heavens* thou wert, and at the voice
Of GOD, as with a mantle, didst invest
The rising *World of Waters*, dark and deep,
Won from the *void* and *formless* Infinite."

Notandùm quòd per *Mithram* intelligebant *Parthi Magi* antiquissimi non *Solem* (ut malè posteri), sed Sole superiorem ; quem appellarunt, Ὁ κρυφιος Θεος, "*Deus absconditus*,"—Παντων Ποιητης και Πατηρ, "*Omnium Opifex et Pater*." *Cudworth*, p. 286. Quibuscum optimè consentit *Job* xxiii. 9. *Judg.* xiii. 18. *Isa.* ix. 5. *Isa.* xlv. 15. Vide § 179.

178. Sana

178. Sana et ſublimis *Noachidarum* Theologia peſſundari et corrumpi incœpit circà quintam à Diluvio generationem; et potiſſimùm à *Nimrod*, *Cuſhitarum* principe, qui cultum aſtrorum primus introduxit; etiàm proverbio celebratus, ut "*Ingens ille venator, coram Domino*;" qui poſt mortem ejus, à ſubditis ſuperſtitioſis, tranſlatus eſt in aſtrum *Orionis* cum canibus (qui *Sirius* et *Canicula* ſunt appellati), quaſi in cœlis etiamnùm venaturus *Urſam Majorem* ejúſque Catulos, Circulo *Arctico* commorantem, et motu diurno revolventem; quam vivus inſectabatur in Montibus *Aſſyriæ*, *Ararat* (*i*): Nam hic bellator acerrimus quoque, cum ab Oriente migraturus ſit in ſedem ab oraculo ſuis deſtinatam, ſubſtitit rebellis " in amœnâ terrâ *Shinaar*, ubi principium dominationis ſuæ poſuit *Babel*, ſeu *Babylonem*; et indè egreſſus, invaſit *Aſſyriam* (ſedem *Noachidarum* poſt Diluvium, et *Shemitæ Peleg*, in diſtributione divinâ terrarum, allocatam), et *Niniven* condidit." Confer *Gen.* x. 8—12. et xi. 1—8. et *Joſephum*.—Colebatur hic *Rebellis* ſub nomine *Bel* (à *Heb.* בעל *Baal*, " Magiſter,") ab Aſſyriis, undè Græcorum *Belus*; cujus nomen proprium forsàn erat *Nin*—(i. e. נין " *Filius*,") (*k*)—utpotè clariſſimus filiorum *Chus*; et quidem appellatur ipſa *Ninive* ab *Herodoto*, paſsìm, ἡ Νινος, " Civitas *Nini*."—Fuit autèm Pronepos *Chus*, per intermedios *Raamah* et

(*i*) Hujuſmodi venationem pulchrè deſcribit *Homerus*, *Il.* xviii. 485.

> Αρκτον δ' ἡν και Αμαξαν επικλησιν καλεουσιν·
> Ἡ τ' αυτο στρεφεται και τ' Ωριωνα δοκευει.
>
> " *Urſámque*, quam cognomine *Plauſtri* appellant [*Ægyptii*],
> Quæ ibidèm ſeſe vertit, obſervátque *Orionem*"

—quaſi illum et canes venaticos metuens.

Atque iterùm *Orionem*, in Infernis, ſimilitèr occupatum introducit, *Odyſſ.* xi. 571.

> Τον δε μετ' Ωριωνα πελωριον εισενοησα
> Θηρας ὁμου ἐιλευντα κατ' ασφοδελον λειμωνα,
> Τους αυτος κατεπεφνεν εν οιοπολοισιν ορεσσιν,
> Χερσιν εχων ῥοπαλον παγχαλκεον αιεν ααγες.
>
> " Poſteà, *Orionem ingentem* aſpexi,
> Feras unà cogentem per aſphodelum pratum,
> Quas *Ipſe* occiderat in montibus deſertis;
> Manibus tenentem clavam prorsùs æneam, ſemper infractam."

Annon facilè derivari poteſt Ωριων à *Chaldæo Uriah* (" Lux Domini"), quæ in Verſione Græcâ *Septuagint.* in caſu obliquo, redditur Ουριαν, 2 *Sam.* xi. 6.?

(*k*) Ingentis hujuſce tyranni nomen proprium fuiſſe נין *Nin*, " filius," cui per odium et ludibrium cognomen indiderunt *Shemitæ* ſubjugati, את נמרד *Æth Nimrod*, " Archirebellem," confirmatur ex *Trogo Pompeio*, ſeu *Juſtini* Lib. I. initio.

" Primus omnium *Ninus*, Rex Aſſyriorum, veterem et quaſi *avitum* gentibus morem, [i. e. *Regimen Patriarchum*,] novâ imperii cupiditate mutavit. Hic primus intulit bella *finitimis*, et rudes adhùc ad reſiſtendum populos, terminos uſque *Libyæ*, perdomuit."—" Dòmitis igitùr proximis, cum acceſſione virium fortior ad alias tranſiret; et proxima quæque victoria inſtrumentum ſequentis eſſet, totius *Orientis* populos ſubegit. Poſtremùm illi bellum cum *Zoroaſtre* Rege *Bactrianorum* fuit (qui primus

 dicitur

et *Sheba*, adeóque *Pelegi* co-ævus, quintus nimirùm à *Noah*, non tertius, ut hactenùs à Chronologis malè ſupponitur.

179. Ingruenti huic idololatriæ fortitèr ſe oppoſuit *Job*, ſeptimus vel octavus à *Noah*. (Vide *Abulfaragi*, p. 13.)

> " Si aſpicerem *lucem* * [*Solis*] ubi ſplendebat, * יאור
> Aut *Lunam* gloriâ incedentem ;
> Si cor meum in occulto ſeductum eſſet,
> Aut manus mea ab ore oſcularetur ;
> Etiam hoc foret *crimen judiciale*,
> Ita enim mentitus eſſem Deo Supremo." *Job* xxxi. 26.

180. Sed, *damnoſa quid non imminuit dies ?*—Sublimem et ſinceram Patriarcharum *Chaldæorum* atque *Hebræorum* Theologiam—" Spiritum Supremum in *ſpiritu* atque *veritate* ritè venerantium," tandem corrupit et inundavit *Sabianiſmus*, ſeu praviſſima idololatria aſtrorum ; "*fingebant enim idololatræ*," ut benè notat *Newtonus*, *Princip*. p. 529.—" *Solem*, *Lunam* et *Aſtra*, *Animas Hominum*, et alias *Mundi partes*, eſſe *partes* Dei Summi ; et ideò *colendas*, ſed falsò.—Atque ex cunctis Philoſophis Mythologíſque *Gentilibus*, eheu !

> " *Apparent rari nantes in gurgite vaſto*,"

qui de Deo ritè ſentiebant.

181. Operam Philoſophis noſtris *Britannis* haùd ingratam me impenſurum ſpero, ſi curſìm ex *Græcis Romaníſque* Mythologis, miram et plorandam ſanè cœcutiam atque vecordiam paucis exemplis illuſtrem, de *Deo materiali* per vocem Αιθηρ, vel *Æther*, ab utríſque intellecto, turpiſſimâ Priſcæ Theologiæ depravatione.

Tragœdorum Græcorum antiquiſſimus, *Æſchylus*, *Ætherem*, eſſe *Deum* apertè poſuit.

dicitur *Artes Magicas* [ſeu diſciplinas *Magorum*] inveniſſe, et *mundi principia*, *ſiderúmque motus ſpectâſſe*.) Hoc occiſo, et ipſe deceſſit ; relicto impubere adhùc filio *Ninya* et uxore *Semiramide*—"

Hìc, *Juſtinus*, more *Græciæ* mendacis, vera falſis immiſcuit. *Nimrodum* probè refert, finitimos invadentem, atque interficientem ultimò, paulò antè obitum, *Zoroaſtrem*, (forsàn *Peleg* ipſum, in Montibus *Ararat*,) Archimagum, peritiſſimúmque Aſtronomum. Sed errore planè ridiculo, ob inſcitiam Linguarum Orientalium, *Trogus* tranſtulit *urbem* Νινευ à *Nimrod* conditam, ſeu *Niniven*, in *filium* "*Ninyam !*"—Et de *Semiramide* fabulatur ! Vera enìm Semiramis, quinque tantùm generationibus, teſte *Herodoto*, anteceſſit *Nitocrem*, inſignem Reginam, quæ Babyloni exſtruendo atque ornando ultimam manum impoſuit ; illa autèm videtur fuiſſe magni *Nebuchadnezzari* uxor, de quo *Daniel*, v. 10.

Secundùm emendationem noſtram *Chronologiæ Sacræ*, incidit diſtributio terrarum habitabilium inter Colonias Noachidarum, " in diebus *Peleg*," (i. e. " *Diviſionis*,") ubi floruit ille *Ninus* Archirebellis, anno 541 circitèr, poſt diluvium ; ſeu anno 2615 antè Æram Chriſtianam : floruit autèm illa *Semiramis* quæ mœnia *Babylonis* conſtruxit, 166 annis ſaltèm antè obitum *Nebuchadnezzar*, ſive 166 + 561 = 727 antè Æram Chriſtianam. *Semiramis* proindè pro uxore *Nabonaſſari*, (cujus æra incepit 747 annis antè Chriſt.) jure haberi poteſt.

ΖΕΥΣ

ΖΕΥΣ εϛιν ΑΙΘΗΡ, Ζευς δε Γη, Ζευς δ' Ουρανος, Ζευς τοι Παντα.

" Jovis est ÆTHER; *Jovis* item *Terra*, et *Jovis Cœlum*, *Jovis* deníque Omnia."

182. Audi quoque *Hippocratis* commentum, *De Princip.* § 1.

Δοκει δε μοι ὁ καλεομεν ΘΕΡΜΟΝ, αθανατον τε ειναι και νοεειν παντα, και ὁρην και ακεειν και ειδεναι παντα, και τα οντα και τα εσομενα· τετο εν, το πλειϛον, ὁτε εταραχθη παντα εξεχωρησεν ες των ανωτατω περιφορην· και ονομηναι μοι αυτο δοκεεσιν οἱ παλαιοι ΑΙΘΕΡΑ.

Ἡ δευτερα μοιρα, ἡ κατωθεν αυτη, καλεεται μεν ΓΗ, ψυχρον και ξηρον και πολυ κινεν· και εν τετεω ενη δη πολυ τε Θερμε. Ἡ δε τριτη μοιρα, ἡ και τε ΗΕΡΟΣ χωριον ειληφε, θερμον τι και ὑγρον εον. Ἡ δε τεταρτη, ἡ τε εγγυτατω προς τη Γη, ὑγροτατον τε και παχυτατον.

" Mihi quidèm videtur Id quod CALIDUM vocamus (Gallicè CALORÍQUE) esse *immortale*; atque omnia *percipere* et *videre* et *audire*, omniáque *ſcire* tam *præſentia* quam *futura*. Hoc igitùr, maximâ ex parte, omnibus perturbatis [*chaoticè*], exceſſit in orbem altiſſimum; ídque nominâſſe mihi videntur Priſci ÆTHEREM."

" Secunda pars, eáque inferior, vocatur TERRA. Eſtque *frigidum* et *ſiccum* et *permobile*; in hâc quoque ineſt *calidi* multum. Tertia autèm pars, quæ *aëris* regionem ſortiebatur, eſt quoddam *calidum* ſimùl et *humidum*. Pars verò quarta, terræ proxima, eſt *humidiſſimum* ſimùl et *denſiſſimum*."

183. Et convenientèr *Virgilius*, *Æn.* VI. 724.

" Principio *Cœlum* et *Terras*, *Campóſque liquentes*,
Lucentémque globum *Lunæ* Titaniáque *Aſtra*,
SPIRITUS intùs alit; totámque, infuſa per artus,
MENS agitat molem, et magno ſe corpore miſcet."

Altiùs hìc ſonant " *Spiritus* et *Mens*;"—ſed ſenſu prorsùs *materiali*, intelligenda; ut ex ſequentibuś patebit, *Georg.* II. 340.

" Cum primum *lucem pecudes* hausêre, *virûmque*
Ferrea progenies duris caput extulit arvis,
Immiſſaéque *Feræ* ſylvis, et *Sidera* cœlo."

184. Rationem reddente *Ovidio*, *Metam.* I. 75.

" Neu regio foret ulla *ſuis animantibus* orba,
Aſtra tenent *Cœleſte Solum*, *Formaéque Deorum*,
Terra Feras cepit."———

185. Pleniùs

S

185. Pleniùs autèm et luculentiùs, *Virgilio, Georg.* IV. 219.

"His quidam ſignis, atque hæc exempla ſecuti,
Eſſe *apibus* partem DIVINÆ MENTIS, et HAUSTUS
ÆTHEREOS, dixêre; DEUM námque ire per omnes
Terráſque, tractúſque *Maris*, *Cælúmque* profundum.
HÌNC, *Pecudes*, *Armenta*, *Viros*, genus omne *Ferarum*,
Quemque ſibì tenues *naſcentem* arceſſere vitas:
Scilicet HÙC reddi deinde, ac reſoluta referri
Omnia; nec *Morti eſſe locum*; ſed *Viva* volare
Sideris in numerum, atque alto ſuccedere *Cælo*."

186. Et apertiſſimè quidem, *Georg.* II. 323.

"Tùm PATER OMNIPOTENS, fœcundis imbribus ÆTHER,
Conjugis in gremium lætæ deſcendit, et omnes
Magnus alit, magno commixtus corpore, *fœtus*."

Hìc procùl dubio, "*Pater Omnipotens*" et *Æther*, pro ſynonymis habentur à *Virgilio*; ſicùt à cæteris paſsìm Poetis *Latinis*,—JUPITER et ÆTHER.

187. Paritèr *Horatius, Od.* I. xxii. 19. et I. i. 25.

"Quod latus mundi, *Nebulæ*, MALUSQUE
JUPITER *urget*."

"———Manet ſub JOVE FRIGIDO
Venator."———

188. Paritérque *Lucanus*:

"JUPITER eſt quodcunque *vides*, quocunque *moveris*."

189. Atque identitatem *Mentis Divinæ* et *Humanæ* apertiſſimè tradiderunt Philoſophi, referente *Cicerone*:—"Sive *anima* ſive *ignis* ſit animus (*humanus*) cum jurarem eſſe *divinum*."—"Et quidem ſi *Deus* aut *anima* aut *ignis* eſt, idem eſt animus hominis:"—"Ergò *animus*, ut ego dico, *divinus* eſt; ut *Euripides* audet dicere, *Deus*."

Et ex hominum motu libero argumentum fabricârunt pro animi æternitate: ut docet quoque *Cicero*:—"Quòd *ſe ipſum moveat*, quis eſt qui hanc naturam *animis* eſſe tributam neget? *Inanimum* enìm eſt omne quod *pulſu* agitatur *externo*; quod autèm eſt *animal*, id *motu* cietur *interiore* et *ſuo*.—Sentit igitùr animus *ſe moveri*; quod cum ſentit, illud ſentit, ſe *vi ſuâ*, non alienâ, *moveri*; nec *accidere poſſe* ut ipſe unquam à ſe deſeratur; ex quo efficitur *æternitas*." (Vid. § 193.)—"Ex DIVINITATE *animos* hauſtos habemus."

190. Talia

190. Talia ſunt dogmata "*Inſanientis Sapientiæ*"—CREATOREM cum *creatis*, *homines* cum *beſtiis*, *animantia* cum *inanimatis*, confundentis; SPIRITUI SUPREMO, exiſtentiam *materialem* impiè tribuentis, *immortalitatem* innatam *animorum hominum pecudúmque* falsò prædicantis; DEUM OPTIMUM MAXIMUM, (cujus eſt "*dare* et *adimere*," vitam commodare et vitam perdere, creare atque annihilare, ſolo ſui ipſius numine, et liberrimo arbitrio, nullo cogente *Fato* ſeu *Neceſſitate cæcâ*,) ingratè prætermittentis et turpitèr dehoneſtantis!

191. Optimè igitùr et acutiſſimè *Paulus* de Philoſophis Gentilibus:—*Rom.* i. 22. (*l*).

Φασκοντες ειναι ΣΟΦΟΙ, ΕΜΩΡΑΝΘΗΣΑΝ.

"Autumantes ſe eſſe SAPIENTES, facti ſunt STULTI."

(*l*) Quantum autèm inſipuerunt, et quàm miſerè cæcutierunt, vel ævo *Auguſti* literarum culturâ celebrato, Philoſophi *Romani*, uno alteróve exemplo oſtendere libet:

Gravem illam, ſublimem et magnificam, DEI SUMMI deſcriptionem *Horatianam* quis non miratur humaniorum litterarum ſcientiâ vel levitèr imbutus? Od. III. 4, 42.

——"Scimus ut impios
Titanas, immanémque turmam
Fulmine ſuſtulerit caduco,
Qui *terram inertem*, qui *mare* temperat
Ventoſum; et *urbes*, *regnáque triſtia*,
Divóſque, *mortaléſque* turbas,
Imperio regit UNUS *æquo*."

Petuntur hæc ſanè ex *Philoſophiâ primâ*, et manifeſtò reſpiciunt *Sodomi* atque *Gomorræ* excidium, *Gen.* xix. 24.; aut *Gigantes* forsàn, *Gen.* vi. 4. Diluvio peremptos. Sed quàm indigna proximè ſequuntur?

"*Magnum* illa *terrorem* intulerat JOVI,
Fidens *Juventus* horrida brachiis,
Fratréſque tendentes opaco
Pelion impoſuiſſe Olympo!"

Quàm incongrua, quàm abſurda, quàm impia cadit concluſio?—Quinque mortales juvenes nempè "*terrorem intuliſſe* DEO SUMMO," ne cœlum ſcandant ipſúmque à ſolio detrudant!!!—Nónne in hoc caſu Poetarum *Ethicorum* princeps (ut fertur *Horatius*), factus eſt ſtultiſſimus?

Et quis feret vel ipſum *Ciceronem* praviſſimè philoſophantem de *perjurio*, quaſi à NUMINE SUMMO prorsùs neglecto atque impunito? *De Officiis*, III. § 28.

"Quid eſt igitùr, dixerit quis, in *jurejurando*? *Num iratum timemus* JOVEM? At hoc quidem COMMUNE EST OMNIUM PHILOSOPHORUM (non eorum modò, qui *Deum nihil habere ipſum negotii, et nihil exhibere alteri* [ut *Epicureorum*]; ſed eorum etiàm qui, *Deum ſemper agere aliquid et moliri* volunt [ut *Stoicorum*, *Peripateticorum*, &c.]) NUNQUAM NEC IRASCI DEUM NEC NOCERE."

Nónne Sophiſtæ tales, (ſicut Cicero *Epicureos* incuſat),

"DEUM *nomine ponunt*, *re tollunt*?"

O Atque

Atque eò ſtultitiæ et vecordiæ progreſſi ſunt, ut NUMEN SUMMUM, paſsìm, per vocem TO ΘΕΙΟΝ deſignârunt, cujus ſignificatio communis apùd Veteres, fuit "*Sulphur Sacrum,*" Anglicè "*Lightning.*"

Sic *Lucas* Caſum *Sodomi* referens, xvii. 29. ex *Geneſi*, xix. 24.

Εβρεξε πυρ και θειον, (Κυριος) απ᾽ ουρανȣ.

"Depluit (DOMINUS) *ignem* et *ſulphur* de cœlo."

Et phraſin claſſicam, Θειȣ πληγ͂ο, exponit *Heſychius*, τȣ κεραυνιȣ πυρος λεγει· οδμην γαρ εχει ὁ κεραυνος θειȣ,—"*Ignem fulmineum* vult; habet enìm *fulmen ſulphuris odorem.*" Appellatur autèm Θειον—το εκ Θεȣ αφιγμενον, utpote "ex DEO *proventum.*"

192. Notatu digniſſimum eſt quòd unico tantùm Evangelii loco, uſurpatur TO ΘΕΙΟΝ in ſenſu philoſophico, pro DIVINITATE ſeu NUMINE SUMMO, ídque à *Paulo*, hallucinationem *Stoicorum* atque *Epicureorum* Athenis perſtringente:—

"Minimè reputare debemus, auro vel argento, vel lapidi, ſculpturæ nimirùm artis ſolertiaéque *humanæ* (TO ΘΕΙΟΝ) DIVINITATEM eſſe ſimilem."—*Acts* xvii. 29.

Omnibus aliis in locis vox Θειον in ſenſu communi adhibetur, pro ſulphure ſacro.

193. Quanto acumine refellit *Paulus* dogmata *Ariſtotelica* de *motu neceſſario*, ſeu *mechanico*:—

Τις ἡ της κινησεως αρχη τῃ ψυχῃ; δηλον δε ὡσπερ εν τῳ ὁλῳ ΘΕΟΣ, και παν εκεινῳ· Κινει γαρ πως τα παντα το εν ἡμιν ΘΕΙΟΝ.—Ἑν τι των αδυνατων το ὑπαρχειν ψυχῃ κινησιν.

"Quid eſt principium motûs *animo?* Patet ſanè, ſicùt in toto DEUS, ità quoque Omne in Illo: *Movet* enìm quodammodo omnia *in nobis* DIVINITAS."—"Ex impoſſibilibus unum eſt quoddam, *animo competere motum.*"

Contrà afferit Apoſtolus, *motum* hominum *voluntarium*, et tamèn à DEO oriundum:—Εν ΑΥΤῼ (ΘΕῼ) γαρ ζωμεν και κινȣμεθα και εσμεν—"In ILLO (DEO) enìm *vivimus* et *noſmet movemus* et *ſumus.*"—Nam κινȣμεθα vox media eſt.—Quâ ratione autèm *noſmet ſponte movemus*, mortalibus ſcire non fas eſt; à DEO hæc facultas conceditur, ſed more nobis prorsùs incognito, et nunquàm forsàn cognoſcendo. Vide § 189.

194. Sufficiant hæc de *Æthere*, ſeu De *Deo Sulphureo* et *Materiali* Sophiſtarum Veterum, hoc noſtro *ſæculo rationis* (ut dicitur) infauſto, eheu! renaſcentium. Annon poeta noſter Ethicus quoque, *Pope*, inſcius forsàn, in hunc turpiſſimum errorem

errorem *Stoicorum* et *Peripateticorum*, à *Cartesio* et *Leibnitzio* malè recoctum, lapsus esse videur, ubi *Deum* hallucinans *Animam Mundi* refinxerit?

> " All are but *parts* of one stupendous *whole*,
> Whose *body* NATURE is, and GOD the *soul*."

Quanto meliùs fuisset, si à disquisitionibus spinosissimis et *periculosæ* plenis *aleæ*, quæ poëtis (qui plerumque in hísce studiis severioribus parùm sunt versati) minimè conveniunt, prorsùs abstinuisset, dogmatis illius sani à se-ipso alibì prolati ritè memor!

> " A *little learning* is a *dangerous* thing;
> *Drink deep*, or *taste not* the PIERIAN SPRING."

Vide § 207.

195. Regerent hìc forsàn Poetæ, *Popiique* fautores, " nónne ipse *Newtonus* paritèr quoque peccat, ubi de *Sensorio* DEI differit in sublimi argumento de existentiâ ENTIS SUPREMI ex operibus Naturæ causísque finalibus patefacti?" *Optics*, p. 345.

" Does it not appear from *phænomena* that there is a BEING *incorporeal*, *living*, *intelligent*, *omnipresent*, who in infinite space (as it were in *his Sensory*) sees the *things themselves* intimately, and thoroughly perceives them, and comprehends them wholly by their immediate presence to HIMSELF: of which things the *images* only, carried through the organs of sense into our *little sensoriums*, are there seen and beheld by *that* which in us perceives and thinks?"

But how cautiously, how correctly does *Newton* guard against misrepresentation of his meaning, in a subsequent passage, p. 379.

" Such a wonderful *uniformity* in the *planetary* system—the *uniformity* in the bodies of *animals*—the *first contrivance* of these very artificial parts—the eyes, ears, brain, muscles, &c. and other organs of sense and motion; and the *instinct* of brutes and insects, can be the *effect* of nothing else than the *wisdom* and *skill* of a *Powerful*, *Everliving* AGENT; who, being in all places, is more able by *his will* to move the bodies within his boundless uniform *sensorium*, and thereby to form and reform the parts of the Universe, than we are by *our will* to move the parts of our own bodies.

" And yet, we are not to consider " *the World* as the *body* of GOD, or the several parts thereof as the *soul* of GOD:"—HE is an UNIFORM BEING, void of organs, members or parts; and *They* are his *creatures*, subordinate to HIM, and subservient to *his will*: and HE is no more *the soul of them*, than the *soul of man* is the soul of the *species* of things carried through the organs of sense into the place of it's sensorium; where *It* perceives them by means of it's immediate presence, without the intervention of any *third* thing.

[N. B.] " The *Organs of Senfe* are not for *enabling the foul to perceive* the fpecies of things in it's fenforium; but only for *conveying* them thither: and God has no need of fuch organs; He being every where prefent to the things themfelves."

DE ENTE SUPREMO.

196. Audiamus *Newtonum* quoque, tunc temporis grandævum, graviffimè et ornatiffimè de hoc myfterio maximè incomprehenfibili differentem, in *Scholio generali Principiorum*, nunquàm fatìs laudando.

" Hic omnia regit, non ut *Anima Mundi*, fed ut *Univerforum Dominus*. Et proptèr dominium fuum Παντοκράτωρ (id eft, *Imperator Univerfalis*), dici folet: Nam *Deus* eft vox relativa, et ad *fervos* refertur; et *Deitas* eft *Dominatio Dei*, non in corpus proprium, (utì fentiunt quibus Deus eft *anima mundi*), fed in fervos.—Vox *Deus* pafsìm fignificat *Dominum* (*m*). *Dominatio* Entis Spiritualis, Deum conftituit, vera verum, fumma fummum, ficta fictum. Et ex *dominatione*

De Nominibus Dei Primitivis.

(*m*) *Newtonus*, " *Pocock* noftrum fecutus, vocem Latinam *Dei* deducit à voce Arabicâ *Du*, (et in obliquo cafu, *Di*), quæ *Dominum* fignificat,"—At minùs rectè, ut videtur. Et forsàn, haùd importunum erit, pro munere meo *profefforio*, quamvis *emerito*, de facrofanctis Dei nominibus primitivis, cautè, verecundè et circumfpectè ex rationibus *etymologycis*, fanioribus magífque fcientificis, forsàn, quam hactenùs per *Lexica* quævis *Orientalia* vel *Inftitutiones Hebræas* prolatis, brevitèr differere.

῾ΑΓΙΑΣΘΗΤΩ ΤΟ ΟΝΟΜΑ ΣΟΥ.

" *Sanctificetur Nomen Tuum.*"

Innuit *Pocock*, nimio Arabifmi ftudio illectus, Linguam *Arabicam* effe *primævam*.—Cæterùm omnium matrem effe *Hebræam*, fatìs conftabit ex nominibus propriis, *Virorum*, *Regionum*, *Fluviorum* et *Urbium*, tàm antè diluvium quàm per quinque generationes poft; quæ omnia funt purè *Hebræa*, et plurima ex cæteris dialectis primitivis exulant. Et neceffariò, quidèm, fi Lingua *Noachidarum* erat eadem ac *Antediluvianorum*; et fi " *Totus Mundus fuit unius linguæ uniúfque dialecti*" ufque ad *Difperfionem Babelitican* et *Linguarum Confufionem*. *Gen.* xi. 1.

Vox Latina *Deus* rectiùs deduci poteft immediatè à Græcâ Δευς; Doricè pro Ζευς,—(*Hefych.*)—tranfmutatione literarum *cognatarum* per omnes gentes ufitatiffimâ; atque hæc iterùm à voce Phœniciâ Ιευω, qualem pronunciabant nomen *Dei* tetragrammatum apud *Hebræos* fanctiffimum, יהוה, *Jeuhōh*; (*Scaliger*, ex *Sanchoniathone*, De *Emend.* Temp. *Fragm.* Not. p. 37.) fimili tranfmutatione, et pro terminatione Hebræâ וה, *ōh*, fubftituendo quoque terminationem Σ, à Linguâ *Medâ*, feu Perficâ primitivâ, mutuatam; (*Herodotus*, I.) Antiquitùs autèm, ab Oraculo *Clarii Apollinis*, pronunciabatur Ιαω, quafi *Jahōh*, fcifcitantibus " *quis Deorum habendus fit qui vocatur* Ιαω?" (*Macrob.* I. 18.)—Undè Prifcorum Græcorum Ζας et Ζαν—Ζανος, et Ionicè Ζην—Ζηνος—(*Hefych.*) et

dominatione verâ sequitur DEUM VERUM esse *vivum*, *intelligentem* et *potentem*: ex reliquis perfectionibus *summum* esse vel *summè perfectum*.

"Deus est UNUS ET IDEM DEUS, *semper et ubique*. *Omnipresens* est, non per *virtutem* solam, sed etiàm per *substantiam*: nam virtus sine substantiâ subsistere non poteſt. In Ipso *continentur* et *moventur* Universa, sed sine mutuâ passione: Deus nihil patitur ex *corporum motibus*; illa nullam sentiunt resistentiam ex *Omnipræsentiâ* DEI.

"Deum

Priscorum Italorum, *Ianus*, Deorum antiquissimus, qui "*Saliorum* quóque antiquissimis carminibus *Deorum Deus* canitur." *Macrob.*

Hanc vocem Ιαω contractiùs pronunciabatur Ιῶ, quam malè confundunt omnes ferè editores Græci atque Latini, cum interjectione O! ut *Euripid. Bacchæ*, 583.

Ιω, Ιω, Δεσποτα, Δεσποτα.

"*Io, Io, Domine, Domine.*"

—pro Ιω Βακχε, "*Io Bacche*," *Hor.* Sat. I. 3. 7.

Itémque *Virgilius*, archaismi observantissimus, *Æn.* 10. 17.

"*Io-pater, Io*—Hominum Divúmque *æterna potestas*."

—quæ vulgò *Jupiter*, pronunciatur, at in casibus obliquis, *Io-vis*, *Io-vi*, &c. verum suum etymon declarat. Vocem Ιῶ interpretatur Hesychius, *ενι*, *uno*; et confirmatur ex *Homeri* Ionicâ, in phrasi Ιῶ ἡματι, *Iliad*, 6. 422. interpretante Scholiâ, *εν μιᾳ ἡμερᾳ*, "*uno die*." Optimè autèm consentit Sacra Scriptura, *Deut.* vi. 4. et *Mark* xii. 29.

"*Audi Israel, Dominus Deus noster Dominus* UNUS *est*."

Et notatu dignissimum est quod nunquàm ab Hebræis יהוה, Ιαω, nunquàm à Græcis, Ζας aut Ζευς, *pluralitèr* usurpantur, utpote *unitatem Dei* designantes.

Ἑις Ζευς, ἑις Ἁδης, ἑις Ἡλιος, ἑις Διονυσος.

"*Unus Jovis, unus Pluto, unus Sol, unus Bacchus*."

Ut refert *Orpheus*, citante *Macrobio*, qui cum *Oraculo Clario* consentit.

Et vox ipsa יהוה ultimò deduci videtur à radice primævâ יה *Iah*; quæ nomen Dei antiquissimum apud Hebræos fuit, (sicùt apud Italos, *Ianus*); et cujus propria significatio optimè forsàn ab *Homero* asservatur in phrasi Ια γηρυς, *Iliad* 4. 437. interpretante Scholio, *μια και αυτη φωνη*, "*una eadémque lingua*."—Et optimè respondet hæc significatio, *Isa.* xxvi. 4. ubi יה et יהוה simùl occurrunt:

"*Fidite Domino usque in perpetuum,*
Quia in (*Iah*, *Iahoh*) *uno eodémque Domino* est *Petra Sæculorum*."

Egregiè igitúr designat radix יה Divinam *immutabilitatem* et *æternitatem*. Confer *Malach.* iii. 6. *Jam.* i. 17. *John* xiv. 1. *Heb.* xiii. 8.

2. A voce Hebræâ, די *Di*, quæ propriè significat *Sufficientiam*, *Copiam*, *Opes*, et undè epitheton divinum שדי *Sadi*,—"*cujus* est *sufficientia*," *Gen.* xvii. 1. et 2 *Cor.* iii. 5. quasi ab radice exsurgunt Arabicâ, *Sadi*; Græca Δις, Δι-ος, quæ *Deum Summum* designabat; cum voce Ζευς synonomans, et sibi posteà casus obliquos Δι-ος, Δι-ι, Δι-α, assumens; et Latina *Dis*, *Di-tis*, Deum Infernum, *Plutonem*, turpiore Theologiæ Primitivæ depravatione; et indè quoque, *Dii Manes*, seu *Dæmones*, mortuorum umbræ, quas pravissimâ superstitione colebant gentes.—Hìnc *Jupiter* appellatur *Dis-piter* ab Horatio, distinctionis gratiâ, non *Diespiter*, ut imperitè editores.

3. A voce Hebræâ אלוה *Ælōh*, quæ propriè ὁ Δυναστης vel Δεσποτης, "*Dominus*," vel "*Imperator*" significat, 1 *Tim.* vi. 15. vel *Jude* iv. Atque DEO SUMMO antiquitùs à gentibus omnibus tributa

"Deum Summum *necessariò existere*, in confesso est. Et eâdem necessitate *semper* est et ubique. Unde etiàm totus est SUI-SIMILIS: Totus oculus, totus auris, totus cerebrum, totus brachium, totus vis sentiendi, intelligendi et agendi; sed more minimè humano, *more nobis prorsùs incognito*. Ut cæcus non habet ideam colorum, sic nos ideam non habemus modorum quibus Deus *Sapientissimus* sentit et intelligit omnia.

"*Corpore omni* et *figurâ corporeâ* prorsùs destituitur; ideóque videri non potest, nec tangi, nec sub specie rei alicujus corporeæ coli debet. Ideas habemus *attributorum* ejus, sed quid sit alicujus rei *substantia* minimè cognoscimus. Videmus tantùm corporum figuras et colores, audimus tantùm sonos, tangimus tantùm superficies externas, olfacimus odores solos, et gustamus sapores; intimas substantias nullo sensu, nullâ actione reflexâ cognoscimus; et multò minus ideam habemus substantiæ Dei.

"HUNC *cognoscimus* solummodò per *proprietates* ejus et *attributa*, et per *sapientissimas* et *optimas* rerum *structuras*, et *causas finales*; et *admiramur* ob *perfectiones*;

tributa est, ritè deducitur variabilis *Allah*, *Ullah* vel *Alo*, quæ vel hodiè cunctas ferè regiones Orientales peragravit; atque ipsa אלוה *Ælōh*, pari analogiâ, ex nomine Dei longè antiquissimo omnium, אל *Æl*, facilè derivatur; quæ abstractè significat ἡ δυναμις, "*Potentia*," concretè verò, ὁ Δυνατος, "*Potens*."—Et notio Dei primaria et omnium simplicissima et usitatissima erat *Potentiæ*, sive beneficæ sive maleficæ. Vid. *Gen.* xxxi. 29. ubi minitatur *Laban Jacobo* genero suo profugo—"Est *potentiæ* manus meæ te *nocere*,"—לאל. Vide quoque 1 *Sam.* xvii. 26. et 2 *Kings*, v. 7. et *Rom.* i. 20. ubì *Paulus* pro parallelis recenset DEI SUMMI Δυναμις και Θειοτης, "*Potentia* et *Deitas*,"—quem probè sequitur *Newtonus*—"*Deitas est Dominatio*."

4. Glissante verò Idololatriâ, malè haberi incœpit vox אל *Æl*, pro *Sole*, numinibúsque fictis Mythologiæ Veterum; unde Solis epitheton, Αελ-ιος, Ηλ-ιος, vel Ἡλ-ιος, in diversis Græciæ (in Theologiâ mendacissimæ) dialectis. Sic plorat *Phaëthon*, fulmine ictus. *Euripid.*

Ω χρυσοφεγγες Ἡλι', ὡς μ' απωλεσας;
Ὀθεν Σ' Απολλων' εμφανως κλησει βροτος.

"O aureâ tædâ prædite *Sol*, quomodo me *perdidisti*!
Undè Te *Perdentem* clarè vocant mortales."

Atque inde pluralitèr usurpari cœperunt אלים et אלהים *Ælim* et *Ælohim*, pro *Angelis*, *Divis*, *Judicibus*, *Heroibus*, &c.—Unde ad retinendam præstantiam DEI SUMMI, cum epithetis variis usi sunt Hebræi Prophetæ, אל אלים, *Æl Ælim*, *Dan.* xi. 36. "*Deus Divorum*;"—אל אלהים, *Æl Ælohim*, "*Deus Deorum*;"—יהוה אלהים, *Iahoh Ælohim*, "*Dominus Deorum*;"—et אל אלהים יהוה *Æl Ælohim Iahōh*, sicùt plenissimè describitur Nomen Dei sacrosanctum, *Josh.* xxii. 22. *Ps.* l. 1. "*Dominus Deus Deorum*."

5. Atque hìnc, ellipsi frequentissimâ אלהים *Ælohim*, licet vox pluralis, usurpatur singularitèr: ut *Gen.* i. 1. ברא אלהים *Deus creavit*; subaudito vel יהים *Iahōb*, vel אל *Æl*, vel utroque. Atque hoc pacto Scripturis Hebræis grammaticè amovetur *solæcismus* iste, qui mirè torsit interpretes, atque varia absurda, imò impia, commenta, *Rabbinorum* et *Mysticorum* peperit; ut videre licet in Lexicis Scriptísque *Kimchi*, *Buxtorf*, *Parkhurst*, &c. &c. &c.

6. Ex his igitùr nominibus Dei sacrosanctis, omnium vetustissimis atque maximè radicalibus, אל *Æl*, et יה *Iah*, utpote simplicissimis, (unde deducuntur Dei nomina propria in omnibus Linguis primitivis, necnon in dialectis, tàm Orientalibus quàm Occidentalibus;) vel ex his solis non inficiabuntur Etymologi æqui et periti, omnium matrem, ipsámque primævam linguam, fuisse *Hebræam*. Sed hæc paucis exponi non possunt.

tiones;

tiones; veneramur autèm et *colimus* ob *dominium:* colimus enìm ut servi, et Deus sine *dominio*, *providentiâ* et *causis finalibus*, nihil aliud est quàm *Fatum* et *Natura:* à cæcâ *Necessitate Metaphysicâ*, quæ utique eadem est semper et ubique, nulla oritur *rerum variatio*: tota rerum conditarum *diversitas* ab *ideis* et *voluntate* Entis necessariò Existentis solummodò oriri *potuit*.

"Dicitur autèm Deus per allegoriam, *videre*, *audire*, *loqui*, *ridere*, *amare*, *odio habere*, *cupere*, *dare*, *accipere*, *gaudere*, *irasci*, *pugnare*, *fabricare*, *condere*, *construere:* nam sermo omnis de Deo, à *rebus humanis* per similitudinem aliquam desumitur, non perfectam quidem, sed *aliqualem* tamen.

"Et hæc de Deo; de quo utique ex *phænominis* differere ad Philosophiam Naturalem pertinet."

197. In hoc *Scholio generali* summè *Theologico*, objiciunt Metaphysici *Leibnitziani*, Newtonum denegare existentiam *spatii* et *durationis* absolutam, ubì asserit, p. 528:

"Deus non est *æternitas* et *infinitas*, sed *æternus* et *infinitus*; non est *duratio* et *spatium*, sed *durat* et *adest:* durat *semper* et adest *ubique*; et *existendo semper et ubique*, *durationem et spatium*, [*æternitatem et infinitatem*] *constituit*."

198. Cæterùm responsio facilis: Si non existeret Deus, quo pacto spatium et duratio ab ullo *Ente intelligente cernerentur?* Nam, ut ipsi loquuntur Metaphysici, *de non apparentibus et de non existentibus eadem est ratio.*—*Newtono* adstipulatur sublimis illa Theologia Patriarcharum et Evangelistarum: docuit enìm *Orpheus*, *Mosem* secutus, quod "*Deus est Lux*—*Lucem habitans inaccessibilem*," et confirmat *Paulus*, in celeberrimâ illâ Dei Summi descriptione, omni laude longè longissiméque majore, quàm potissimùm secutus atque interpretatus est *Newtonus:* 1 *Tim.* vi. 15.

Ὁ ΜΑΚΑΡΙΟΣ ΚΑΙ ΜΟΝΟΣ ΔΥΝΑΣΤΗΣ,
Ὁ ΒΑΣΙΛΕΥΣ ΤΩΝ ΒΑΣΙΛΕΥΟΝΤΩΝ,
ΚΑΙ ΚΥΡΙΟΣ ΤΩΝ ΚΥΡΙΟΝΤΩΝ·
Ὁ μονος εχων αθανασιαν·
Φως οικων απροσιτον·
Ὁν ειδεν ουδεις ανθρωπων, ουδε ιδειν δυναται·
Ὡ ΤΙΜΗ ΚΑΙ ΚΡΑΤΟΣ ΑΙΩΝΙΟΝ. ΑΜΗΝ.

"Felix atque Solus Imperator,
Rex Regentium et Dominus Dominantium;
Solus habens immortalitatem,
Lucem habitans inaccessibilem,
Quem vidit nemo mortalium, nec videre potest:
Cui Honos et Dominium æternum. Amen."

199. Con-

199. Confirmat quóque *Johannes:*

‘Ο ΘΕΟΣ *φως εςι, και Σκοτια εν Αυτω εκ εςιν εδεμια.*

“ Deus *Lux* eſt, atque in Illo haudquaquàm eſt *caligo.*”

Πνευμα ‘Ο ΘΕΟΣ—‘Ο ΘΕΟΣ *Αγαπη εςι.*

“ *Spiritus,* Deus”—“ Deus eſt *Charitas.*”

At ſi prædicantur *Lux, Spiritus, Charitas,* de Deo *abſtractè,* ut loquuntur Logici, non concretè, nónne pari analogiâ in re prorsùs incognitâ, “ *conſtituat* quoque *durationem* et *ſpatium?*”

200. Ingenti *veræ Theologiæ* detrimento fines *Philoſophiæ Naturalis* (cujus eſt “ fidelitèr Theologiæ ancillari,” ut optimè *Bacon*), temerè tranſgrediuntur Metaphyſici; ubi malè ſani profiteri audent “ *Dei exiſtentiam et attributa demonſtrari à priori;*”— ut infauſtâ diligentiâ, Doctus *Clarke,* ſpeculationibus metaphyſicis ex controverſiâ *Leibnitzianâ* naſcentibus, nimìs imbutus, et iiſdem periculosè indulgens. Deum enim *neceſſariò exiſtere,* certiſſimum eſt: ſed talis exiſtentiæ idea, utcunque per ſe poſitiva, noſtro conceptui eſt planè *negativa;* ſolummodò indicans “ *nullam cauſam*” ejus dari extrinſecùs: cæterùm *ideæ negativæ ſcientiam poſitivam minimè pariunt,* ipſo teſtante *Locke, Eſſay,* &c. IV. 1. 5.

201. *Exiſtentia Dei* intellectum omnium creaturarum ratione præditarum tranſcendit, imò *Angelorum* et *Archangelorum,* præter “ Filium Charitatis ejus,”—Jesum Christum Dominum nostrum:—qui “ *Illi proximos occupavit honores*”—*Hor.* ut ex myſtagogo maximo, *Johanne,* diſcimus, *Rev.* xix. 16.

ΒΑΣΙΛΕΥΣ ΒΑΣΙΛΕΩΝ
ΚΑΙ ΚΥΡΙΟΣ ΚΥΡΙΩΝ.

“ Rex Regum et Dominus Dominorum.” (*n*)

Cui

(*n*) Quàm accuratè diſtinguunt Myſtagogi omnium maximi, *Paulus* et *Johannes,* inter Θεον τον Πατερα, “ *Deum Patrem,*” atque ‘Υιον τον Μονογενη, “ *Filium unicè Genitum,* ſeu *Dilectum,*” *John,* i. 16. *Paulus* enìm Patrem deſignat titulo pleniſſimo maximéque auguſto, ὁ Βασιλευς των Βασιλευοντων, και Κυριος των Κυριοντων, quem haudquaquàm aſſequitur exilitas Linguæ *Latinæ,* articulo *emphatico,* ὁ, carentis; ſed qui *Anglicè* efferri poteſt—“ The King of the reigning [*Kings*], and Lord of the ruling [Lords.]”—*Johannes* verò Filium, titulo planè proximo: Βασιλευς Βασιλεων, και Κυριος Κυριων—“ King of Kings, and Lord of Lords,”—*uno eodémque* Spiritu Divino ambo afflati.

Nec

Cui inditum eſt Dei gratiâ, Ονομα το ὑπερ παν ονομα, "*Nomen ſuprà omne nomen*" (vel auctoritatem) tùm in *Cælis*, tùm in *Terris*, tùm in *Infernis*,—εις δοξαν ΘΕΟΥ ΠΑΤΡΟΣ, "*in laudem* DEI PATRIS;" nunc, ſub *novo fœdere*, *Phil.* ii. 9; ſicùt olìm, ſub *primo fœdere*, *Exod.* xxiii. 21.—Nam aſſerit ipſe 'Ο ΛΟΓΟΣ ΤΟΥ ΘΕΟΥ, "*Oraculum Dei*:" *Matt.* xi. 27.

Παντα ΜΟΙ παρεδοθη ὑπο ΤΟΥ ΠΑΤΡΟΣ,
Και ὀυδεις επιγινωσκει ΤΟΝ 'ΥΙΟΝ, ει μη 'Ο ΠΑΤΗΡ·
Ουδε ΤΟΝ ΠΑΤΕΡΑ τις επιγινωσκει, ει μη 'Ο 'ΥΙΟΣ,
ΚΑΙ ᾧ εαν βουληται 'Ο 'ΥΙΟΣ αποκαλυψαι.

"*Omnia* MIHI *commiſſa ſunt à* PATRE;
Et nemo *intimè cognoſcit* FILIUM, niſi PATER:
Neque PATREM quis *intimè cognoſcit*, niſi FILIUS,
Et cuicunque *voluerit* FILIUS *revelare*."

Nec *Ethnicos* latuit hæc diſtinctio: Audi *Horatium*.

"*Quid priùs dicam ſolitis* PARENTIS
Laudibus, qui res Hominum ac Deorum,
Qui Mare et Terras, variiſque Mundum
Temperat horis?
Unde nil majus generatur IPSO:
Nec viget quicquam ſimile aut ſecundum.
Proximos ILLI *tamen occupavit*
PALLAS *honores*."

Hæc certè ex *Theologiæ primæ* reliquiis extraxit *Horatius*.—"PARENS" enìm eſt ὁ Πατηρ Ανδρωντε Θεωντε, *Orphei*, *Homeri* et *Heſiodi*, licet quid hâc appellatione maximè honorificâ verè ſignificaretur parùm callebant: PALLAS verò, gentium *Dea Bellatrix*, à Παλλω, "*vibro*," [*fulmen, telum*, &c.] derivatur; hæc enìm vox *epitheton* tantùm deſignat—Παλλας Αθηνη, ut *Homero* paſsìm. Ejus autèm nomen Αθηνη ab Egyptiacâ Νηϊθ, ſeu SAPIENTIA, retrò legendo derivatur; ut *Not.* (*p*) monuimus: quæ eadem primitùs fuit cum דבר יהוה, *Dabar Iahoh* Teſtamenti Veteris, *Gen.* xv. 1. ſeu Novi—'Ο ΛΟΓΟΣ ΤΟΥ ΘΕΟΥ, "*Oraculum Dei*," *Rev.* xix. 13.—ΕΚΕΙΝΟΣ εξηγησατο—"ILLE *expoſuit*."—*John*, i. 18.

Notatu digniſſimum eſt quòd, referente *Horatio*, PALLAS, non *arripuit*, non *arrogavit* ſibi, ſed DEI gratiâ, "*Proximos* ILLI *occupavit* honores,"—*occupavit*, quaſi *jure hæreditario*; ſicùt optimè exponit *Paulus*, de JESU CHRISTO—'Ος εν μορφη Θεου ὑπαρχων, ουχ' ἁρπαγμον ἡγησατο το ειναι Ισα-Θεω: αλλ' ἑαυτον εκενωσε, &c.—"Qui in *formâ divinâ* [licet] ſubſiſtens, non *inſtar prædæ* duxit *fieri Deo ſimilem*; ſed ſe-ipſum *exhauſit*," &c.—*Phil.* ii. 7.

Fabulantur Mythologi Ethnici, DEAM SAPIENTIAM panopliâ inſtructam εκ Διος κεφαλης, "*è Jovis capite*" emicuiſſe, plenè adultam, non infantem; temerè confundentes, ut videtur, vocem Hebræam ראש *Raoſh*, ambiguè ſignificantem tùm *caput* tùm *principium*, in ſublimi illâ deſcriptione ortûs *Sapientiæ primævæ*, *Prov.* viii. 22.

"DOMINUS *genuit* ME, *principium viæ ſuæ*,
Ante opera ſua, primitùs:
Ab æternitate ordinatus ſum, à capite,
A primordiis Terræ."———

Ubi phraſis Hebræa lineæ penultimæ, מראש *Meraŏſh*, quæ ad literam redditur, "*à capite*," ſeu "*ex capite*," æquipollet phraſi lineæ primæ, ראשית דרכו *Raŏſh derecbu*—"*principium viæ ſuæ*;"—quam optimè interpretatur *Johannes*, *Rev.* iii. 14.—'Η αρχη της κτισεως Θεου, "*Primordium creationis Dei*," ſeu πρωτοτοκος πασης κτισεως, "*Primogenitus omnis creationis*," *Coloſſ* i. 15.—De JESU CHRISTO. Et pari ambiguitate, phraſin *Orphei*, Ζευς κεφαλη, mutat *Plutarchus* in Ζευς αρχη. *Not.* (*g*).

202. Non nisi *stultissimi* igitùr autumant se *à priori* de DEO SUMMO philosophari *Cartesiani*, *Leibnitziani*, *Clarkiani* et *Unitarii*; quorum idcircò—"*dubitans, circumspectans, hæsitans, multa adversa revertens, tanquàm in rate in mari immenso, omnis vehitur oratio*,"—(ut plorat *Cicero* de *Mythologis* antiquis):—"*Relegens errata retrorsùm littora*"—*Virgil.*—"*Insanientis dum Sapientiæ consulti errant*;—et *Retrorsùm vela dare, atque iterare cùrsus coguntur relictos.*" *Hor.*

"*Quantò satiùs est ire apertâ viâ et rectâ* [THEOLOGIÆ REVELATÆ], *quàm* TIBI IPSI *flexus disponere*; *quos cum magnâ molestiâ debeas relegere?*" *Seneca.*

203. Gravissiméque perstringit *Paulus* hallucinationem inexcusabilem veterum philosophantium (*o*): *Rom.* i. 19.

Διοτι το γνωστον ΤΟΥ ΘΕΟΥ *φανερον εστιν εν αυτοις*· 'Ο *γαρ* ΘΕΟΣ *εφανερωσε αυτοις* (*τα γαρ αορατα* Αυτȣ, *απο κτισεως* Κοσμȣ, *τοις ποιημασι νοȣμενα καθοραται*· 'Ητε *αιδιος* Αυτȣ ΔΥΝΑΜΙΣ *και* ΘΕΙΟΤΗΣ) *εις το ειναι αυτȣς αναπολογητȣς.*

"Quocircà, quicquid *scibile* de DEO, manifestum est inter illos: DEUS enìm manifestabat illis (nam ab usque mundo condito, [*attributa*] ejus *invisibilia*;—nempè ÆTERNA ejus POTENTIA et DEITAS,—*operibus intelligenda*, cernere erat), ut illi forent *inexcusabiles.*"

204. Paritérque *Johannes* *ακαταληψιαν* Mundi docet:

> Και ΤΟ ΦΩΣ *εν τη Σκοτια φαινει,*
> Και 'Η *Σκοτια αυτο ȣ κατελαβεν.*

> "Et LUX in *Caligine* lucet;
> Sed *Caligo* eam *non comprehendebat.*"

Etsì gestiebant Veteres, docente *Paulo*: *Act.* xvii. 27.

> *Ζητειν* ΤΟΝ ΚΥΡΙΟΝ, *ει αρα ψηλαφησειαν*
> Αυτον, *και ἑυροιεν*·

> —"*Quærere* DOMINUM, si fortè *palparent* Illum,
> Atque invenirent."

205. Testante itèm *Socrate* (Græcorum sapientissimo, secundùm *Oraculi Delphici* effatum, quòd, *se nihil scire præter suam inscitiam de rebus divinis*, professus est), in *Phædone*, undè *Paulus* proculdubiò excerpsit:

(*o*) Turpissimè hallucinatur vel ipse *Aristoteles*, referente *Cicerone*, *Nat. Deorum*, II. 16. qui, ex *ordine* et *constantiâ* motuum cœlestium, astris ipsis *sensum* atque *intelligentiam* stultissimè tribuit!

—"*Ordo* autem astrorum, et in *omni æternitate constantia*, neque *naturam* significat, ut enim plena rationis; neque *fortunam*, quæ, amica *varietati*, constantiam respuit: sequitur ergò, ut ipsa *suâ sponte, suo sensu*, ac *divinitate* moveantur."—

"Cœlestium ergò *admirabilem ordinem, incredibilémque constantiam*, ex quâ *conservatio* et *salus* omnium omnis oritur, qui *vacare mente* putat, is ipse *mentis expers* habendus est." *Nat. Deor.* II. 21.

To

Το γαρ μη διελεσθαι οἱον τ' ειναι, ὁτι αλλο μεν τι εςι το αἰτιον τω ονῖι, αλλο δ' εκεινο ανευ ἑ το αἰτιον ϐκ αν ποτ' ειη αἰτιον· ὁ δη μοι φαινονται ψηλαφωντες οἱ πολλοι, ὡσπερ εν σκοτει αλλοτριω ομματι προσχρωμενοι, ὡς αἰτιον αυτο προσαγορευειν.

"Nam diſtinguendi non capax eſſe, quod unum quidem eſt *ipſa rei alicujus exiſtentis cauſa*, aliud verò, *illa, ſine quâ non foret unquam cauſa:* quoad hoc ſanè, mihi videntur plerique, *palpantes quasì in caligine*; alienis oculis uſi, ſiquidèm *cauſam ipſam,* [immediatam, ultimam falsò] appellant." (*p*).

206. Optiméque et luculentiſſimè quidèm exponit *Paulus*, argumentum illud acutiſſimum omniúmque inviĉtiſſimum quo *Newtonus*, ex "*rerum variatione*," et *tempeſtatum diverſitate*, NUMEN SUPREMUM colligit: *Aĉt.* xiv. 17.

(*p*) Præclarum quaſi *Socratis* teſtimonium de hallucinatione veterum philoſophorum DEI SUMMI naturam inveſtigare ſatagentium, affert *Lucianus*, in *Halcyone*, Vol. I. p. 179.

Ω φιλε Χαιρεφων, εοικαμεν ἡμεις των δυνατων τε και αδυνατων ΑΜΒΛΥΩΠΟΙ τινες ειναι κριται παντελως. Δοκιμαζομεν γαρ δη, κατα δυναμιν ανθρωπινην, [ΤΗΝ ΔΥΝΑΜΙΝ] ΑΓΝΩΣΤΟΝ· ϐσαν και ΑΠΙΣΤΟΝ και ΑΟΡΑΤΟΝ· Πολλα ϐν φαινεται ἡμιν και των ευπορων απορα, και των εφικτων ανεφικτα· συχνα μεν και δι' απειριαν, συχνα δε και δια νηπιοτητα φρενων· τω οντι γαρ νηπιος εοικεν ειναι πας ανθρωπος, και ὁ πανυ γερων, επειτα μικρος πανυ και νεογιλος ὁ τϐ βιϐ χρονος, προς τον παντα αιωνα. Τι δαν, ὦ 'γαθε, οἱ αγνοϐντες τας ΤΩΝ ΘΕΩΝ και ΔΑΙΜΟΝΩΝ ΔΥΝΑΜΕΙΣ, εχοιεν αν ειπειν ποτερον δυνατον η αδυνατον τι των τοιϐτων.

"O chare Chærophon, *Nos* quidèm *tàm poſſibilium quàm impoſſibilium* videmur eſſe *Judices* HALLUCINANTES prorsùs. Æſtimamus enìm, *ſecundum potentiam humanam*, [POTENTIAM] NON SCIBILEM, INCREDIBILEM, INVISIBILEM: Multa igitùr ex facilibus difficilia, et ex attingendis non attingenda, nobis apparent. Crebrò quidèm *propter imperitiam*, crebrò autèm *propter infantiam mentis*: Reverà enìm *Infans* videtur eſſe *omnis homo*, etiamſi valdè ſenex; quoniam exiguum prorsùs et puſillum tempus eſt vitæ, cum omni ævo comparatum. Quâ ratione autem, O bone! poſſint, qui neſciunt DEORUMDÆMONÚMQUE POTENTIAS, dicere *utrum talium aliquid ſit poſſibile vel impoſſibile?*"

Atque hinc optimè exponenda Inſcriptio Altaris *Athenis*, *Aĉt.* xvii. 23.

ΑΓΝΩΣΤΩ ΘΕΩ.

"NON SCIBILI DEO."

Inſcriptio forsàn derivanda ab Inſcriptione Templi *Nëith* ſeu *Sapientiæ* apud *Sais* in Ægypto Superiore, unde primitùs Colonia *Athenis* deducta fuerat. Et forsàn vox *Athenæ* (Αθηνα), ab Ægyptiâ Νηϊθ vel Νηϊθας, ſi ordine retrogrado legatur, optimè deduci poteſt; ſicut conjecit doctiſſimus *Cudworth*, *Intelleĉt. Syſt.* p. 309, ex *Platone* in *Timæo*, referens:

'Οτι της πολεως [Σαις] Θεος αρχηγος εςιν Αιγυπτιςι μεν τϐνομα Νηϊθ, Ἑλληνιςι δε, ὡς ὁ εκεινων λογος, Αθηνα.

"Urbis [*Sais*] *Dea præſul*, eſt *Nëith*, nomine Ægyptiaco; *Athena* autem, Græco; ſicùt eorum ſermo."

Inſcriptio *Saitica*, a *Plutarcho* ſervatur:

Εγω ειμι ΠΑΝ, το ΓΕΓΟΝΟΣ, και ΟΝ, και ΕΣΟΜΕΝΟΝ.
Και τον εμον πεπλον ϐδεις πω Θνητος απεκαλυψεν.

"*Ego ſum* OMNE, *quod* FUIT, *quod* EST, et *quod* ERIT:
Atque meum peplum nullus adhuc Mortalis revelavit."

Atqui nemo quidem Mortalium revelare potuit, donec Ipſe demiſſus eſt e ſinu PATRIS, ὁ μονογενης, "*unicè dileĉtus* FILIUS, qui naturam cultúmque DEI cæcutientibus mortalibus exponeret." *John* i. 18.

Καιτοιγε

Καίτοιγε ουκ αμαρτυρον Ἑαυτον αφηκεν (Ὁ ΘΕΟΣ) αγαθοποιων, ουρανοθεν ἡμιν ὑετους διδους και καιρους καρποφορους, εμπιπλων τροφης και ευφροσυνης τας καρδιας ἡμων.

"Veruntamèn, se ipsum *haud intestatum* relinquebat (Deus) utique *benefaciendo*, cœlitùs *imbres* nobis ac *tempestates fructuosas præbendo*, corda nostra *cibo* atque *lætitiâ* implendo."

Haud raró enìm omnia fiunt contraria: puniuntur immemorum atque ingratorum corda, ultione divinâ, regionum siccitate, tempestatibus infructuosis, fame et pavore palpitantium:

> "*Terra enim ferax in salsuginem convertitur,*
> *Propter iniquitatem incolentium ejus.*"

Atqui hoc argumentum ex *causis finalibus* petitum, omnium conceptibus accommodatissimum, vix, et ne vix quidem, attigerunt Sophistæ veteres; dum *Legibus Naturæ generalibus*, secundùm quas ordinantur anni cardines tempestatúmque vicissitudines revolutionibus periodísque certis et æquabilibus, *mechanicè* quasi agentibus, tantùm incumbentes, CAUSÆ PRIMARIÆ obliti, in pravissimum *Atheismum* delapsi sunt.—Vide *Aristotelem*, astris (propter ordinem et constantiam motuum quibus in cœlis circumferri observantur,) sensum et intelligentiam afferentem.—*Nat. Deor.* II. 16.

"*Extat igitur*, ut præclarè loquitur *Cotesius*, "*eximium* NEWTONI OPUS *adversùs* ATHEORUM *impetus munitissimum præsidium:* neque enim alicunde feliciùs, [ORACULIS DIVINIS *exceptis*], *quàm ex hâc pharetrâ, contrà impiam catervam tela depromseris.*"—*Præfat.* ad secundam editionem Principiorum Newtoni, anno 1713 à Cotesio factam.

207. Coronidis loco subjungere libet sententiam auream cedróque dignam, quam protulit magnus ille BACON, *Philosophiæ Experimentalis Pater*, et NEWTONI *præceptor:*

"A *superficial taste* of PHILOSOPHY may perchance incline the mind to ATHEISM; but a *full draught* thereof brings it back again to RELIGION.

"In the entrance of Philosophy, when the *second causes*, most obvious to the senses, offer to the mind, we are apt to cleave unto them, and dwell too much upon them; so as to *forget* what is superior in Nature: but when we pass further, and behold the *dependency* and *confederacy of causes*, and the *works* of PROVIDENCE; then, according to the Poets, we easily perceive, that *the highest link of* NATURE'S *chain must be tied to the foot of* JUPITER'S *chair—that* PHILOSOPHY, *like* JACOB'S *vision, discovers to us a* LADDER, *whose top reaches up to the footstool of the throne of* GOD."

HINC OMNE PRINCIPIUM, HUC REFER EXITUM.

Typis J. DAVIS, in vico dicto Chancery-Lane.